A NEW FAMILY OF RATIO-CUM-PRODUCT ESTIMATORS OF POPULATION MEAN FOR TWO CONCOMITANT VARIABLES UNDER EXTREME RANKED SET SAMPLING (ERSS)

By

Etorti, Imoke John [1]
Department of Mathematics, Cross River State College of Education, Akamkpa –Nigeria[1]
imoke.etorti@crs-coeakamkpa.edu.ng
+2348062220171

Effanga Effanga Okon [2]
eoeffanga@yahoo.com
+2347036507635

Mbe E. Nja[3]
mbe_nja@yahoo.com
+2347061614812

Department of Statistics, University of Calabar, Calabar-Nigeria [2, 3]

Abstract
When observations are costly and time consuming but the ranking of the observations without actual measurement can be done with ease comparatively, ERSS can be employed instead of Simple Random Sampling (SRS), to gain more information for estimation purposes. In this paper and in an attempt to address the problem of loss of efficiency usually suffered in estimation of population mean under SRS, a new family of ratio-cum-product estimators of population mean of the study variable Y is proposed based on ERSS using information on two concomitant variables. Members of the proposed family of estimators were obtained by varying the values of the scalars that aid in developing the estimators. Various properties of the estimators such as biases, relative biases, Mean Square Errors (MSEs), and Optimal Mean Square Errors (OMSEs) were derived to the quadratic form of Taylor's series approximation. Empirical study was conducted using three natural population data sets in order to investigate the performances and efficiency of the proposed family of estimators under ERSS over its corresponding counterpart's estimator based on SRS and some existing ratio and product estimators. This empirical study was followed up with a computer simulation study using R-software. The results revealed that the proposed family of estimators in ERSS produced about 50% smaller MSEs which is an indicator of appreciable gain in efficiency and superiority over its corresponding counterpart estimator and some existing ratio type estimators in sample survey for all cases considered in this paper and were therefore adjudged to provide a better alternative whenever efficiency is required. **(Word count 255)**

Keywords: Extreme Ranked Set Sampling, Mean Square Errors, Simple Random Sampling, Ratio-cum-product Estimator, simulation.

1.1 Introduction

Ranked set sampling (RSS) is an approach to dealing with sample selection. It was proposed in the seminal paper of McIntyre (1952). His experience in agricultural application provoked a challenge to the usual simple random sampling (SRS) design introducing a previous ordering of the units. The practical studies suggested that it produces more accurate estimators of the mean. His proposal was taken into account by other researchers dealing with agricultural studies. They also obtained better results using RSS. The mathematical validity of the McIntyre's instinctive postulation was sustained by the work of Takahasi and Wakimoto (1968) on unbiased estimates of the population mean based on sample stratified by means of ordering. That fact also remained unnoticed by the majority of the statistical community but some interesting results were developed for establishing the mathematical reasons sustaining having a better result when using RSS. Dell and Clutter (1972) proved that deviations in ordering lowers the accuracy of the RSS mean comparative to SRS mean. Nevertheless, RSS mean is always superior over the SRS mean till ordering is too substandard as to produce a random sample when its performances is akin to that of SRS mean.

The techniques of Extreme-Ranked Set Sampling (ERSS) was first introduced by Samawi *et al.* (1996) to estimate the population mean and showed that the mean based on ERSS though unbiased but is more efficient that the sample mean due to SRS. Furthermore, Samawi (1996) introduced the principle of Stratified Ranked Set Sampling (SRSS); to improve the precision of estimating the population means in case of SSRS.
Haq and Shabbir (2010) suggested a family of ratio estimator for population mean in extreme ranked set sampling using two auxiliary variables and illustrated that the estimators under ERSS are more efficient in comparison to estimator based on SRS especially when the underlying population is symmetric.

Al-Omari (2019) developed and improved ratio-cum-product estimators of the population for single concomitant variable under ERSS and SRS motivated by the Singh and Espejo (2003) ratio-cum-product estimators and showed that ERSS techniques provides a better and improved results in comparison with SRS sampling procedure.
Ali and Iqbal (2021) proposed an efficient generalized family of estimators to estimate finite population mean of study variable under Ranked Set Sampling utilizing information on an auxiliary variable and concluded that when correlation between the study and auxiliary variables increases, the proposed generalized family of estimators proved to be efficient estimator of population mean of the study variable.

Imoke et al. (2022) suggested a class of ratio-cum-product estimators for population mean following information on a single accompanying variable, under ERSS and SRS techniques and successfully showed that the suggested class of estimators in ERSS produced smaller biases and MSEs which is an indicator of appreciable gain in efficiency and superiority over its corresponding counterpart estimator and some existing ratio type estimators in sample survey for all cases considered in paper and were therefore adjudged to provide a better alternative whenever efficiency is required.

Other researchers who worked on RSS and its modifications include but not limited to Al-Omari el tal.(2009), Kaur et al. (1995) Al Saleh and Al-Kadiri (2000), Al-Saleh and Al-Omari (2002), Abu Dayyeh et al. (2002), Al-Saleh and-Zheng(2002), Al-Saleh and Samawi (2000), Ozturk and Wolfe (2000), Ozturk (2002), Al-Saleh and Ababneh (2015), Zheng and Al-Saleh (2002), Al-Saleh and Darabseh (2017)). In continuation of the search for a better method of estimating the population mean, this paper put forward a new family Ratio-cum product estimators for population mean under Extreme Ranked Set Sampling (ERSS) that would evaluate properties such as biases, relative biases, Mean Square Errors (MSEs) to a degree desired when compared to its corresponding counterpart estimator based on SRS and some existing ones. Analytical and simulation study of performances and efficiencies of the estimators over the usual SRS method using their (MSEs) were carried out in an attempt to support the theoretical results with numerical illustration, from where conclusion was drawn following the results obtained from the paper.

2.0 Sampling methods

Here, we present the sampling scheme which are employed in the course of this paper i.e Extreme Ranked Set Sampling (ERSS), as well as the frequently used (SRS).

2.1 Extreme Ranked Set Sampling (ERSS)

The ERSS method, as suggested by Samawi et al. (1996), can be described as given below:

a: Select m random samples, each of size m units, from an infinite population and order the units within each sample with respect to a variable under consideration by impressionistic method or any other cost-free procedure. For exact quantification, if the sample size m is even, from the first $\frac{m}{2}$ sets, select the smallest ordered units and from the other $\frac{m}{2}$ sets select the largest ranked unit. Such a sample shall be represented by ERSSe.

b: If the sample size m is odd, then there are two options:

(i) From the first $\frac{m-1}{2}$ sets we choose the average of the observation of the smallest units in the $\frac{m-1}{2}$ sets, and from the other $\frac{m-1}{2}$ sets, we take the average of the measures of the largest ranked unit. Such a sample shall be represented by $ERSS_{0(a)}$.

(ii) From the remaining measure of the m^{th} unit we take the median. Such a sample will be represented by $ERSS_{0(m)}$.

e: is even

$0(a)$: is odd average

$0(m)$: is odd median

The procedure can be continued r times, if need be, to get a sample of size rm units. The choices of (a) and $b(ii)$ is usually less difficult in application than the choice of $b(i)$. In this paper, we considered the choices of (a) and $b(ii)$, (i.e the even case and the case of taking the median from the m^{th} sample if m is odd).

Let $(X_{i(1)},\ Y_{i[1]}, Z_{i(1)})$, $(X_{i(2)},\ Y_{i[2]}, Z_{i(2)})$, …, $\left(X_{i(m)},\ Y_{i[m]}, Z_{i(m)}\right)$, be the ordered Statistics of the i^{th} sample $(X_{i(1)},\ Y_{i[1]}, Z_{i(1)})$, $(X_{i(2)},\ Y_{i[2]}, Z_{i(2)})$, , …, $\left(X_{i(m)},\ Y_{i[m]}, Z_{i(m)}\right)$, $(i = 1, 2, \dots, m)$. If m is even, then:

$(X_{1(1)},\ Y_{1[1]}, Z_{1(1)})$, $(X_{2(1)},\ Y_{2[1]}, Z_{2(1)})$, $\left(X_{3(1)},\ Y_{3[1]}, Z_{3(1)}\right)$,…,$(X_{m-1(1)}$, $Y_{m-1[1]},\ Z_{m-1(1)})$, $\left(X_{m(m)},\ Y_{m[m],}\ Z_{m(m)}\right)$ represent ERSSe.

The estimator of the means and variances using ERSSe of sample size m (recall that m is even) is defined by:

$$\left.\begin{aligned}\bar{X}^{ERSSe} &= \frac{1}{2}\left(\bar{X}_{(1)}, + \bar{X}_{(m)}\right) = \frac{2}{m}\left(\sum_{i=1}^{\frac{m}{2}} X_{2i-1(1)} + \sum_{i=1}^{m} X_{2i(m)}\right)\\ \bar{Y}^{ERSSe} &= \frac{1}{2}\left(\bar{Y}_{(1)}, + \bar{Y}_{[m]}\right) = \frac{2}{m}\left(\sum_{i=1}^{\frac{m}{2}} Y_{2i-1[1]} + \sum_{i=1}^{m} Y_{2i[m]}\right)\\ \bar{Z}^{ERSSe} &= \frac{1}{2}\left(\bar{Z}_{(1)}, + \bar{Z}_{(m)}\right) = \frac{2}{m}\left(\sum_{i=1}^{\frac{m}{2}} Z_{2i-1(m)} + \sum_{i=1}^{m} Z_{2i(m)}\right)\end{aligned}\right\} \quad (1)$$

$$\left.\begin{aligned}E(\bar{X}^{ERSSe}) &= \frac{1}{m}\left(\mu_{x(1)}, + \mu_{x(m)}\right)\\ E(\bar{Y}^{ERSSe}) &= \frac{1}{m}\left(\mu_{y[1]}, + \mu_{y[m]}\right)\\ E(\bar{Z}^{ERSSe}) &= \frac{1}{m}\left(\mu_{z(1)}, + \mu_{z(m)}\right)\end{aligned}\right\} \quad (2)$$

with variances

$$\left.\begin{aligned}Var(\bar{X}^{ERSSe}) &= \frac{1}{2m}(\sigma^2_{X(1)} + \sigma^2_{X(m)})\\ Var(\bar{Y}^{ERSSe}) &= \frac{1}{2m}(\sigma^2_{Y[1]} + \sigma^2_{Y[m]})\\ Var(\bar{Z}^{ERSSe}) &= \frac{1}{2m}(\sigma^2_{Z(1)} + \sigma^2_{Z(m)})\end{aligned}\right\} \quad (3)$$

If m is odd, then using $ERSS_{0(m)}$ the measures

$(X_{1(1)},\ Y_{1[1]}, Z_{1(1)}),(X_{2(1)},$

$Y_{2[1]}, Z_{2(1)}), (X_{3(1)},\ Y_{3[1]}, Z_{3(1)}),\ldots(X_{m-1(1)},\ Y_{m-1[1]}, Z_{m-1(1)}), \Big[X_{m\left(\frac{m+1}{2}\right)},$

$Y_{m\left[\frac{m+1}{2}\right]}, Z_{m\left(\frac{m+1}{2}\right)}\Big]$ represents $ERSS_{0(m)}$

$$\bar{X}^{ERSS_{0(m)}} = \frac{X_{1(1)} + X_{2(1)} + X_{3(1)} + \ldots + X_{m-1(m)} + X_{m\left(\frac{m+1}{2}\right)}}{m},$$

$$\bar{Y}^{ERSS_{0(m)}} = \frac{Y_{1[1]} + Y_{2[2]} + Y_{[3]} + \ldots + Y_{m-1[m]} + Y_{m\left[\frac{m+1}{2}\right]}}{m},$$

$$\bar{Z}^{ERSS_{0(m)}} = \frac{Z_{1(1)} + Z_{2(2)} + Z_{(3)} + \ldots + Z_{m-1(m)} + Z_{m\left(\frac{m+1}{2}\right)}}{m},$$

we can check easily that

$$\left.\begin{aligned}E(\bar{X}^{ERSS_{0(m)}}) &= \frac{m-1}{2m}\left(\mu_{x(1)}, + \mu_{x(m)}\right) + \frac{1}{m}\mu_{x\left(\frac{m+1}{2}\right)}\\ E(\bar{Y}^{ERSS_{0(m)}}) &= \frac{m-1}{2m}\left(\mu_{y[1]}, + \mu_{y[m]}\right) + \frac{1}{m}\mu_{y\left[\frac{m+1}{2}\right]}\\ E(\bar{Z}^{ERSS_{0(m)}}) &= \frac{m-1}{2m}\left(\mu_{z(1)}, + \mu_{z(m)}\right) + \frac{1}{m}\mu_{z\left(\frac{m+1}{2}\right)}\end{aligned}\right\} \quad (4)$$

Using the fact that $X_{1(1)}, X_{2(1)}, X_{3(1)}, \ldots, X_{m-1(m)},\ Y_{1[1]}, Y_{2[1]}, Y_{3[1]}, \ldots, Y_{m-1[m]}$ are all independent and $Z_{1[1]}, Z_{2[1]}, Z_{3[1]}, \ldots, Z_{m-1(m)}$ are all independent, the variance of $\bar{X}^{ERSS_{0(m)}}$, $\bar{Y}^{ERSS_{0(m)}}, and\ \bar{Z}^{ERSS_{0(m)}}$ can be shown to be:

$$\left.\begin{aligned} Var(\bar{X}^{ERSS_{0(m)}}) &= \frac{(m-1)}{2m^2}\left(\sigma^2_{X(1)} + \sigma^2_{X(m)}\right) + \frac{1}{m^2}\sigma^2_{x(\frac{m+1}{2})} \\ Var(\bar{Y}^{ERSS_{0(m)}}) &= \frac{(m-1)}{2m^2}\left(\sigma^2_{Y[1]} + \sigma^2_{Y[m]}\right) + \frac{1}{m^2}\sigma^2_{y[\frac{m+1}{2}]} \\ Var(\bar{Z}^{ERSS_{0(m)}}) &= \frac{(m-1)}{2m^2}\left(\sigma^2_{Z(1)} + \sigma^2_{Z(m)}\right) + \frac{1}{m^2}\sigma^2_{z(\frac{m+1}{2})} \end{aligned}\right\} \quad (5)$$

2.2 Simple Random Sampling

In case of two concomitant variables X and Z, when ranking is done on Z.

For simplification of notation, we will assume that for $\left(X_{i(j)},\ Y_{i[j]},\ Z_{i(j)}\right)$ and according to our description, $(X_{11},\ Y_{11}, Z_{11})$, $(X_{21},\ Y_{21}, Z_{21})$, …, $(X_{m1},\ Y_{m1}, Z_{m1})$ is the SRS with mean

$$\left.\begin{aligned} \bar{X}^{SRS} &= \frac{1}{m}\left(\sum_{i=1}^{m} X_i\right) \\ \bar{Y}^{SRS} &= \frac{1}{m}\left(\sum_{i=1}^{m} Y_i\right) \\ \bar{Z}^{SRS} &= \frac{1}{m}\left(\sum_{i=1}^{m} Z_i\right) \end{aligned}\right\} \quad (6)$$

with variances

$$\left.\begin{aligned} Var(\bar{X}^{SRS}) &= \left(\frac{1-f}{m}\right)\sigma_x^2 \\ Var(\bar{Y}^{SRS}) &= \left(\frac{1-f}{m}\right)\sigma_y^2 \\ Var(\bar{Z}^{SRS}) &= \left(\frac{1-f}{m}\right)\sigma_z^2 \end{aligned}\right\} \quad (7)$$

if the finite population correction $f \neq 0$

and

$$\left.\begin{aligned} \rho_{XY} &= \frac{\sigma_{XY}}{\sigma_X\sigma_Y}, \quad \sigma_{XY} = Cov(\bar{Y}^{SRS}, \bar{X}^{SRS}) = \rho_{XY}\sigma_X\sigma_Y \\ \rho_{XZ} &= \frac{\sigma_{XZ}}{\sigma_X\sigma_Y}, \quad \sigma_{XY} = Cov(\bar{X}^{SRS}, Z^{SRS}) = \rho_{XZ}\sigma_X\sigma_Z \\ \rho_{YZ} &= \frac{\sigma_{YZ}}{\sigma_Y\sigma_Z}, \quad \sigma_{YZ} = Cov(\bar{Y}^{SRS}, \bar{Z}^{SRS}) = \rho_{YZ}\sigma_Y\sigma_Z \end{aligned}\right\} \quad (8)$$

2.3 Notations and some useful equations

The following notations and expressions shall be useful in the course of this paper. For all $i = 1, 2, \ldots, m$. in case of two concomitant variables X and Z, when ranking is done on Z.

$$\left.\begin{array}{c} \mu_x = E(X_i) \\ \sigma_x^2 = var(X_i) \\ \mu_{x1} = E\big(X_{i(1)}\big) \\ \mu_{x\left(\frac{m+1}{2}\right)} = E\left(X_{i\left(\frac{m+1}{2}\right)}\right) \\ \sigma_{x1}^2 = var\big(X_{i(1)}\big) \\ \sigma^2_{x\left(\frac{m+1}{2}\right)} = var(X_{i\left(\frac{m+1}{2}\right)} \\ \sigma_{xm}^2 = var\big(X_{i(m)}\big) \\ \sigma_{x(1,m)} = \text{cov}\big(X_{m(1)}, X_{m(m)}\big) \end{array}\right\} \quad (9)$$

$$\left.\begin{array}{c} \mu_y = E(Y_i) \\ \sigma_y^2 = var(Y_i) \\ \mu_{y[1]} = E(Y_{i[1]}) \\ \mu_{y\left(\frac{m+1}{2}\right)} = E\left(Y_{i\left[\frac{m+1}{2}\right]}\right) \\ \sigma_{y1}^2 = var\big(Y_{i1]}\big) \\ \sigma^2_{y\left(\frac{m+1}{2}\right)} = var\left(Y_{i\left[\frac{m+1}{2}\right]}\right) \\ \sigma_{ym}^2 = var\big(Y_{i[m]}\big) \\ \sigma_{y(1,m)} = \text{cov}\big(Y_{m[1]}, Y_{m[m]}\big) \end{array}\right\} \quad (10)$$

$$\left.\begin{array}{c} \mu_z = E(Z_i) \\ \sigma_z^2 = var(Z_i) \\ \mu_{z[1]} = E(Z_{i(1)}) \\ \mu_{z\left(\frac{m+1}{2}\right)} = E\left(Z_{i\left(\frac{m+1}{2}\right)}\right) \\ \sigma_{z1}^2 = var\big(Z_{i(1)}\big) \\ \sigma^2_{z\left(\frac{m+1}{2}\right)} = var(Z_{i\left(\frac{m+1}{2}\right)} \\ \sigma_{zm}^2 = var\big(Z_{i(m)}\big) \\ \sigma_{z(1,m)} = \text{cov}\big(Z_{m(1)}, Z_{m(m)}\big) \end{array}\right\} \quad (11)$$

Remark 2.1

If the underlying distribution is symmetric about the origin 0, then $X_i \cong -\mathrm{X}_{(m-i+1)}$, likewise $Y_i \cong -\mathrm{Y}_{[m-i+1]}$ and $Z_i \cong -\mathrm{Z}_{(m-i+1)}$ Arnold, Balakrishman and Nagaraja (1992) showed that $X_i \cong -\mathrm{X}_{(m-i+1)}$, and $\sigma^2_{Y[1]} = \sigma^2_{X(1)} = \sigma^2_{Z(1)} = \sigma^2_{X(m-i+1)}$ for all $i = 1, 2, \dots, m$. This implies that $\mu_{x(1)} = -\mu_{x(m)}$, $\mu_{y[1]} = -\mu_{y[m]}$, also $\mu_{z(1)} = -\mu_{z(m)}$ and if m is odd, $\mu_{y\left[\frac{m+1}{2}\right]} = \mu_{x\left(\frac{m+1}{2}\right)} = \mu_{z\left(\frac{m+1}{2}\right)} = \mu = 0$ where $\frac{m+1}{2}$ means the median rank. Also $\sigma^2_{Y(1)} = \sigma^2_{X(1)} = \sigma^2_{Z(1)} = \sigma^2_{X(m)}$. Using the above results, $E(\bar{X}^{ERSS\boldsymbol{e}}) = 0$, $E(\bar{X}^{ERSS_{0(a)}}) = 0$, and $E(\bar{X}^{SRS}) = 0$. Therefore equations (3) and (5) will boil down to:

$$\left.\begin{aligned} Var(\bar{X}_0^{ERSSe}) &= \frac{1}{m}(\sigma^2_{X(1)}) \\ Var(\bar{Y}_0^{ERSSe}) &= \frac{1}{m}(\sigma^2_{Y[1]}) \\ Var(\bar{Z}_0^{ERSSe}) &= \frac{1}{m}(\sigma^2_{Z(1)}) \end{aligned}\right\} \quad (13)$$

if the finite population correction $f \to 0$

$$\left.\begin{aligned} Var(\bar{X}_0^{ERSS_{0(m)}}) &= \frac{1}{m^2}\left((m-1)\left(\sigma^2_{X(1)}\right) + \sigma^2_{X(\frac{m+1}{2})}\right) \\ Var(\bar{Y}_0^{ERSS_{0(m)}}) &= \frac{1}{m^2}\left((m-1)\left(\sigma^2_{Y[1]}\right) + \sigma^2_{y[\frac{m+1}{2}]}\right) \\ Var(\bar{Z}_0^{ERSS_{0(m)}}) &= \frac{1}{m^2}\left((m-1)\left(\sigma^2_{Z(1)}\right) + \sigma^2_{z(\frac{m+1}{2})}\right) \end{aligned}\right\} \quad (14)$$

3.0 Some Existing Estimators in SRS with two auxiliary variables with their MSEs

$$T_S = \bar{y}\left(\frac{\bar{X}+\rho_{xz}}{\bar{x}+\rho_{xz}}\right)\left(\frac{\bar{z}+\rho_{xz}}{\bar{Z}+\rho_{xz}}\right)$$

Singh and Taylor (2005)

$$\text{MSE}(T_S)= \left(\frac{1-f}{m}\right)\bar{Y}^2\left(C_y^2+3C_x^2+C_z^2-2C_{xy}+2C_{yz}-2C_{xz}\right) \quad (15)$$

$$T_{S2} = \bar{y}\left(\frac{\bar{x}^*}{\bar{X}}\right)\left(\frac{\bar{Z}}{\bar{z}^*}\right)$$

Sing et al (2011)

$$\text{MSE}(T_{S2})= \left(\frac{1-f}{m}\right)\bar{Y}^2\left(C_y^2+3g^2C_z^2-g^2C_x^2+4gC_{xy}-4gC_{yz}+4g^2C_{xz}\right) \quad (16)$$

$$T_V = \bar{y}\left(\frac{a\bar{x}^*+b}{a\bar{X}+b}\right)^{\delta_1}\left(\frac{a\bar{Z}+b}{a\bar{z}^*+b}\right)^{\delta_2}$$

Vishwakarma and Kumar (2015)

$$\text{MSE}(T_{S2})= \left(\frac{1-f}{m}\right)(\bar{Y}^{SRS})^2\left(C_0^2+3\frac{(\delta_1(\delta_1+1)}{2}\lambda_a^2g^2C_1^2+\frac{\delta_2(\delta_2-1)}{2}g^2\lambda_b^2C_2^2-4g\lambda_a\delta_1C_{10}-4g\lambda_b\delta_2C_{20}+4g^2\delta_1\delta_2\lambda_a\lambda_bC_{21}\right) \quad (17)$$

3.1 The proposed class of estimator based on ERSS

As an extension of Imoke et al. (2022) class of ratio-cum-product estimators of population mean using a single accompanying variable under ERSS and SRS, we proposed a new family of ratio-cum-product type estimator of population mean Y in case of two accompanying variables X and Z, when ranking is done on Z using ERSS techniques as:

$$T_{2Ej} = \bar{y}^{ERSS_e}\left[w_1\left(\frac{a\bar{X}^{ERSS_e}+\rho}{a\bar{x}^{ERSS_e}+\rho}\right)^{\alpha_1}\left(\frac{b\bar{z}^{ERSS_e}+\rho}{b\bar{Z}^{ERSS_e}+\rho}\right)^{\alpha_2}+ w_2\left(\frac{a\bar{x}^{*ERSS_e}+\rho}{a*\bar{X}^{ERSS_e}+\rho}\right)^{\delta_1}\left(\frac{b\bar{Z}^{ERSS_e}+\rho}{b\bar{z}^{*ERSS_e}+\rho}\right)^{\delta_2}\right] \quad (18)$$

$$\boldsymbol{T_{2(0(m))j}} = \bar{y}^{ERSS_{0(m)}}\left[w_1\left(\frac{a\bar{X}^{ERSS_{0(m)}}+\rho}{a\bar{x}^{ERSS_{0(m)}}+\rho}\right)^{\alpha_1}\left(\frac{b\bar{z}^{ERSS_{0(m)}}+\rho}{b\bar{Z}^{ERSS_{0(m)}}+\rho}\right)^{\alpha_2}+ w_2\left(\frac{a\bar{x}^{*ERSS_{0(m)}}+\rho}{a*\bar{X}^{ERSS_{0(m)}}+\rho}\right)^{\delta_1}\left(\frac{b\bar{Z}^{ERSS_{0(m)}}+\rho}{b\bar{z}^{*ERSS_{0(m)}}+\rho}\right)^{\delta_2}\right] \quad (19)$$

where $(a, b \neq 0,\ \rho \neq 0)$ are real numbers and also may take the values of parameters associated with either the study variable y or the accompanying variables (x, z) ; in this case, the coefficient of variation and the correlation coefficient respectively $(\alpha_1,\ \alpha_2,\ \delta_1, \delta_2)$ are scalars or real constants which helps in designing the estimators and can be determined suitably. (w_1, w_2) are suitably chosen scalars whose sum need not be unity. when $\alpha_1,\ \alpha_2$, δ_1, and δ_2 are fixed, w_1, w_2 may be selected in an optimum manner by minimizing the (MSEs) of $T_{2Ej}\ \ j = 1\ to\ m$ with respect to w_1, w_2. Where $\bar{x}^{*ERSS_e} = \{(1+g)\bar{X}^{ERSS_e} - g\bar{x}^{ERSS_e}\}$, $z^{*ERSS} = \{(1+g)\bar{Z}^{ERSS} - g\bar{z}^{ERSS}\}$ are unbiased estimator of population means, $\bar{X}^{ERSS_e}$, $\bar{Z}^{ERSS_e}$ respectively,

$$g = \frac{m}{(M-m)},\ |(1 - g\lambda_i e_i)| < 1,\ or\ \left|\left(1 - g\lambda_i\left(\frac{\bar{x}^{ERSS_e} - \bar{X}^{ERSS_e}}{\bar{X}^{ERSS_e}}\right)\right)\right| < 1$$

$i = 0, 1, 2,$ for all the ${}^{M}C_m$ samples.

Where $\lambda_i = \frac{\bar{X}^{ERSS_e}}{\bar{X}^{ERSS_e} + \rho}$, $e_y = \frac{\bar{y}^{ERSS_e} - \bar{Y}^{ERSS_e}}{\bar{Y}^{ERSS_e}}$, $e_x = \frac{\bar{x}^{ERSS_e} - \bar{X}^{ERSS_e}}{\bar{X}^{ERSS_e}}$, $e_z = \frac{\bar{z}^{ERSS_e} - \bar{Z}^{ERSS_e}}{\bar{Z}^{ERSS_e}}$

3.2 The second proposed class of estimator based on SRS

$$T_{2Si} = \bar{y}^{SRS}\left[w_1\left(\frac{a\bar{X}^{SRS} + \rho}{a\bar{x}^{SRS} + \rho}\right)^{\alpha_1}\left(\frac{b\bar{z}^{SRS} + \rho}{b\bar{Z}^{SRS} + \rho}\right)^{\alpha_2} + w_2\left(\frac{a\bar{x}^{*SRS} + \rho}{a * \bar{X}^{SRS} + \rho}\right)^{\delta_1}\left(\frac{b\bar{Z}^{SRS} + \rho}{b\bar{z}^{*SRS} + \rho}\right)^{\delta_2}\right] \quad (20)$$

where $\bar{x}^{*SRS} = \{(1+g)\bar{X}^{SRS} - g\bar{x}^{SRS}\}$ and $\bar{z}^{*}{}_{ERSS} = \{(1+g)\bar{Z}^{SRS} - g\bar{z}^{SRS}\}$ are unbiased estimators of population mean, $\bar{X}^{SRS}$, $\bar{Z}^{SRS}$ respectively

$$g = \frac{m}{(M-m)} = \frac{f}{(1-f)}\ \text{and}\ f = \frac{m}{M},\ |(1 - g\lambda_i e_i)| < 1,\ or\ \left|\left(1 - g\lambda_i\left(\frac{\bar{x}^{SRS} - \bar{X}^{SRS}}{\bar{X}^{SRS}}\right)\right)\right| < 1$$

$i = 0, 1, 2.$ For all the ${}^{M}C_m$ samples.

TABLE 1
Some members of the class of estimators T_{2Ej}

S.N	Estimators	Values of Scalars								
		w_1	w_2	a	b	ρ	α_1	α_2	δ_1	δ_2
1	$T_{2E1} = \bar{y}^{ERSS_e}$	1	0	0	0	0	0	0	0	0
2	$T_{2E2} = \bar{y}^{ERSS_e}\left(\frac{\bar{X}^{ERSS_e}}{x^{ERSS_e}}\right)$	1	0	1	1	0	1	0	0	0
3	$T_{2E3} = \bar{y}^{ERSS_e}\left(\frac{\bar{z}^{ERSS_e}}{\bar{Z}^{ERSS_e}}\right)$	1	0	0	1	0	0	1	0	0
4	$T_{2E4} = \bar{y}^{ERSS_e}\left(\frac{\bar{x}^{*ERSS_e}}{\bar{X}^{ERSS_e}}\right)$	0	1	1	1	0	0	0	**1**	0
5	$T_{2E5} = \bar{y}^{ERSS_e}\left(\frac{\bar{Z}^{ERSS_e}}{\bar{z}^{*ERSS_e}}\right)$	0	1	0	1	0	0	0	1	1
6	$T_{2E6} = \bar{y}^{ERSS_e}\left(\frac{\bar{X}^{ERSS_e}}{x^{ERSS_e}}\right)\left(\frac{\bar{z}^{ERSS_e}}{\bar{Z}^{ERSS_e}}\right)$	1	0	1	1	0	1	1	0	0
7	$T_{2E7} = \bar{y}^{ERSS_e}\left(\frac{\bar{x}^{*ERSS_e}}{\bar{X}^{ERSS_e}}\right)\left(\frac{\bar{Z}^{ERSS_e}}{\bar{z}^{*ERSS_e}}\right)$	0	1	1	1	0	0	0	1	1
8	$T_{2E8} = \bar{y}^{ERSS_e}\left(\frac{\bar{X}^{ERSS_e}}{x^{ERSS_e}}\right)^{\alpha_1}\left(\frac{\bar{z}^{ERSS_e}}{\bar{Z}^{ERSS_e}}\right)^{\alpha_2}$	1	0	1	1	0	α_1	α_2	0	0
9	$T_{2E9} = \bar{y}^{ERSS_e}\left(\frac{\bar{x}^{*ERSS_e}}{\bar{X}^{ERSS_e}}\right)^{\delta_1}\left(\frac{\bar{Z}^{ERSS_e}}{\bar{z}^{*ERSS_e}}\right)^{\delta_2}$	0	1	1	1	0	0	0	δ_1	δ_2
10	$T_{2E10} = \bar{y}^{ERSS_e}\left(\frac{\bar{X}^{ERSS_e}+\rho}{\bar{x}^{ERSS_e}+\rho}\right)\left(\frac{\bar{z}^{ERSS_e}+\rho}{\bar{Z}^{ERSS_e}+\rho}\right)$	1	0	1	1	ρ	1	1	0	0
11	$T_{2E11} = \bar{y}^{ERSS_e}\left(\frac{\bar{x}^{*ERSS_e}+\rho}{\bar{X}^{ERSS_e}+\rho}\right)\left(\frac{\bar{Z}^{ERSS_e}+\rho}{\bar{z}^{*ERSS_e}+\rho}\right)$	0	1	1	1	ρ	0	0	1	1
12	$T_{2E12} = \bar{y}^{ERSS_e}\left(\frac{\bar{X}^{ERSS_e}+\rho}{\bar{x}^{ERSS}+\rho}\right)^{\alpha_1}\left(\frac{\bar{z}^{ERSS_e}+\rho}{\bar{Z}^{ERSS_e}+\rho}\right)^{\alpha_2}$	1	0	1	1	ρ	α_1	α_2	0	0
13	$T_{2E13} = \bar{y}^{ERSS_e}\left(\frac{\bar{x}^{*ERSS_e}+\rho}{\bar{X}^{ERSS_e}+\rho}\right)^{\delta_1}\left(\frac{\bar{Z}^{ERSS_e}+\rho}{\bar{z}^{*ERSS_e}+\rho}\right)^{\delta_2}$	0	1	1	1	ρ	0	0	δ_1	δ_2
14	$T_{2E14} = \bar{y}^{ERSS_e}\left(\frac{a\bar{x}^{*ERSS_e}+\rho}{a\bar{X}^{ERSS_e}+\rho}\right)\left(\frac{b\bar{Z}^{EERSS_e}+\rho}{b\bar{z}^{*ERSS_e}+\rho}\right)$	0	1	a	b	ρ	0	0	1	1
15	$T_{2E15} = \bar{y}^{ERSS_e}\left(\frac{a\bar{X}^{ERSS_e}+\rho}{a\bar{x}^{ERSS_e}+\rho}\right)^{\alpha_1}\left(\frac{b\bar{z}^{ERSS_e}+\rho}{b\bar{Z}^{ERSS_e}+\rho}\right)^{\alpha_2}$	1	0	a	b	ρ	α_1	α_2	0	0
16	$T_{2E16} = \bar{y}^{ERSS_e}\left(\frac{a\bar{x}^{*ERSS_e}+\rho}{a*\bar{X}^{ERSS_e}+\rho}\right)^{\delta_1}\left(\frac{b\bar{Z}^{ERSS_e}+\rho}{b\bar{z}^{*ERSS_e}+\rho}\right)^{\delta_2}$	0	1	a	b	ρ	0	0	δ_1	δ_2

TABLE 2
Some members of the class of estimators $T_{2(0(m))j}$

S/N	Estimators	Values of Scalars								
		w_1	w_2	a	b	ρ	α_1	α_2	δ_1	δ_2
1	$T_{2(0(m))1} = \bar{y}^{ERSS_{0(m)}}$	1	0	0	0	0	0	0	0	0
2	$T_{2(0(m))2} = \bar{y}^{ERSS_{0(m)}}\left(\frac{\bar{X}^{ERSS_{0(m)}}}{x^{ERSS_{0(m)}}}\right)$	1	0	1	1	0	1	0	0	0
3	$T_{2(0(m))3} = \bar{y}^{ERSS_{0(m)}}\left(\frac{\bar{z}^{ERSS_{0(m)}}}{\bar{Z}^{ERSS_{0(m)}}}\right)$	1	0	0	1	0	0	1	0	0
4	$T_{2(0(m))4} = \bar{y}^{ERSS_{0(m)}}\left(\frac{\bar{x}^{*ERSS_{0(m)}}}{\bar{X}^{ERSS_{0(m)_e}}}\right)$	0	1	1	1	0	0	0	**1**	0
5	$T_{2(0(m))5} = \bar{y}^{ERSS_{0(m)}}\left(\frac{\bar{Z}^{ERSS_{0(m)}}}{\bar{z}^{*ERSS_{0(m)}}}\right)$	0	1	0	1	0	0	0	1	1
6	$T_{2(0(m))6} = \bar{y}^{ERSS_{0(m)}}\left(\frac{\bar{X}^{ERSS_{0(m)}}}{x^{ERSS_{0(m)}}}\right)\left(\frac{\bar{z}^{ERSS_{0(m)}}}{\bar{Z}^{ERSS_{0(m)}}}\right)$	1	0	1	1	0	1	1	0	0
7	$T_{2(0(m))7} = \bar{y}^{ERSS_{0(m)}}\left(\frac{\bar{x}^{*ERSS_{0(m)}}}{\bar{X}^{ERSS_{0(m)}}}\right)\left(\frac{\bar{Z}^{ERSS_{0(m)}}}{\bar{z}^{*ERSS_{0(m)}}}\right)$	0	1	1	1	0	0	0	1	1
8	$T_{2(0(m))8} = \bar{y}^{ERSS_{0(m)}}\left(\frac{\bar{X}^{ERSS_{0(m)}}}{x^{ERSS_{0(m)}}}\right)^{\alpha_1}\left(\frac{\bar{z}^{ERSS_{0(m)}}}{\bar{Z}^{ERSS_{0(m)}}}\right)^{\alpha_2}$	1	0	1	1	0	α_1	α_2	0	0
9	$T_{2(0(m))9} = \bar{y}^{ERSS_{0(m)}}\left(\frac{\bar{x}^{*ERSS_{0(m)}}}{\bar{X}^{ERSS_{0(m)}}}\right)^{\delta_1}\left(\frac{\bar{Z}^{ERSS_{0(m)}}}{\bar{z}^{*ERSS_{0(m)}}}\right)^{\delta_2}$	0	1	1	1	0	0	0	δ_1	δ_2
10	$T_{2(0(m))10} = \bar{y}^{ERSS_{0(m)}}\left(\frac{\bar{X}^{ERSS_{0(m)}}+\rho}{\bar{x}^{ERSS_{0(m)}}+\rho}\right)\left(\frac{\bar{z}^{ERSS_{0(m)}}+\rho}{\bar{Z}^{ERSS_{0(m)}}+\rho}\right)$	1	0	1	1	ρ	1	1	0	0
11	$T_{2(0(m))11} = \bar{y}^{ERSS_{0(m)}}\left(\frac{\bar{x}^{*ERSS_{0(m)}}+\rho}{\bar{X}^{ERSS_e}+\rho}\right)\left(\frac{\bar{Z}^{ERSS_{0(m)}}+\rho}{\bar{z}^{*ERSS_{0(m)}}+\rho}\right)$	0	1	1	1	ρ	0	0	1	1
12	$T_{2(0(m))12} = \bar{y}^{ERSS_{0(m)}}\left(\frac{\bar{X}^{ERSS_{0(m)}+\rho}}{\bar{x}^{ERSS_{0(m)}+\rho}}\right)^{\alpha_1}\left(\frac{\bar{z}^{ERSS_{0(m)}+\rho}}{\bar{Z}^{ERSS_{0(m)}+\rho}}\right)^{\alpha_2}$	1	0	1	1	ρ	α_1	α_2	0	0
13	$T_{2(0(m))13} = \bar{y}^{ERSS_{0(m)}}\left(\frac{\bar{x}^{*ERSS_{0(m)}}+\rho}{\bar{X}^{ERSS_{0(m)}}+\rho}\right)^{\delta_1}\left(\frac{\bar{Z}^{ERSS_{0(m)}}+\rho}{\bar{z}^{*ERSS_{0(m)}}+\rho}\right)^{\delta_2}$	0	1	1	1	ρ	0	0	δ_1	δ_2
14	$T_{2(0(m))14} = \bar{y}^{ERSS_{0(m)}}\left(\frac{a\bar{x}^{*ERSS_{0(m)}}+\rho}{a\bar{X}^{ERSS_{0(m)}}+\rho}\right)\left(\frac{b\bar{Z}^{ERSS_{0(m)}}+\rho}{b\bar{z}^{*ERSS_{0(m)}}+\rho}\right)$	0	1	a	b	ρ	0	0	1	1
15	$T_{2(0(m))15} = \bar{y}^{ERSS_{0(m)}}\left(\frac{a\bar{X}^{ERSS_{0(m)}}+\rho}{a\bar{x}^{ERSS_{0(m)}}+\rho}\right)^{\alpha_1}\left(\frac{b\bar{z}^{ERSS_{0(m)}}+\rho}{b\bar{Z}^{ERSS_{0(m)}}+\rho}\right)^{\alpha_2}$	1	0	a	b	ρ	α_1	α_2	0	0
16	$T_{2(0(m))16} = \bar{y}^{ERSS_{0(m)}}\left(\frac{a\bar{x}^{*ERSS_{0(m)}}+\rho}{a*\bar{X}^{ERSS_{0(m)}}+\rho}\right)^{\delta_1}\left(\frac{b\bar{Z}^{ERSS_{0(m)}}+\rho}{b\bar{z}^{*ERSS_{0(m)}}+\rho}\right)^{\delta_2}$	0	1	a	b	ρ	0	0	δ_1	δ_2

TABLE 3
Some members of the class of estimators T_{2Sj} , $j = 1, 2, 3, \dots 16$

S/N	Estimators T_{2Sj}	Values of Scalars								
		w_1	w_2	a	b	ρ	α_1	α_2	δ_1	δ_2
1	$T_{2S1} = \bar{y}^{SRS}$	1	0	0	0	0	0	0	0	0
2	$T_{2S2} = \bar{y}^{SRS}\left(\frac{\bar{X}^{SRS}}{x^{SRS}}\right)$	1	0	1	1	0	1	0	0	0
3	$T_{2S3} = \bar{y}^{SRS}\left(\frac{\bar{z}^{SRS}}{\bar{Z}^{SRS}}\right)$	1	0	0	1	0	0	1	0	0
4	$T_{2S4} = \bar{y}^{SRS}\left(\frac{\bar{x}^{*SRS}}{\bar{X}^{SRS}}\right)$	0	1	1	1	0	0	0	**1**	0
5	$T_{2S5} = \bar{y}^{SRS}\left(\frac{\bar{Z}^{SRS}}{\bar{z}^{*SRS}}\right)$	0	1	0	1	0	0	0	1	1
6	$T_{2S6} = \bar{y}^{SRS}\left(\frac{\bar{X}^{SRS}}{\bar{x}^{SRS}}\right)\left(\frac{\bar{z}^{SRS}}{\bar{Z}^{SRS}}\right)$	1	0	1	1	0	1	1	0	0
7	$T_{2S7} = \bar{y}^{SRS}\left(\frac{\bar{x}^{*SRS}}{\bar{X}^{SRS}}\right)\left(\frac{\bar{Z}^{SRS}}{\bar{z}^{*SRS}}\right)$	0	1	1	1	0	0	0	1	1
8	$T_{2S8} = \bar{y}^{SRS}\left(\frac{\bar{X}^{SRS}}{x^{SRS}}\right)^{\alpha_1}\left(\frac{\bar{z}^{SRS}}{\bar{Z}^{SRS}}\right)^{\alpha_2}$	1	0	1	1	0	α_1	α_2	0	0
9	$T_{2S9} = \bar{y}^{SRS}\left(\frac{\bar{x}^{*SRS}}{\bar{X}^{SRS}}\right)^{\delta_1}\left(\frac{\bar{Z}^{SRS}}{\bar{z}^{*SRS}}\right)^{\delta_2}$	0	1	1	1	0	0	0	δ_1	δ_2
10	$T_{2S10} = \bar{y}^{SRS}\left(\frac{\bar{X}^{SRS}+\rho}{\bar{x}^{SRS}+\rho}\right)\left(\frac{\bar{z}^{SRS}+\rho}{\bar{Z}^{SRS}+\rho}\right)$	1	0	1	1	ρ	1	1	0	0
11	$T_{2S11} = \bar{y}^{SRS}\left(\frac{\bar{x}^{*SRS}+\rho}{\bar{X}^{SRS}+\rho}\right)\left(\frac{\bar{Z}^{SRS}+\rho}{\bar{z}^{*SRS}+\rho}\right)$	0	1	1	1	ρ	0	0	1	1
12	$T_{2S12} = \bar{y}^{SRS}\left(\frac{\bar{X}^{SRS}+\rho}{\bar{x}^{SRS}+\rho}\right)^{\alpha_1}\left(\frac{\bar{z}^{SRS}+\rho}{\bar{Z}^{SRS}+\rho}\right)^{\alpha_2}$	1	0	1	1	ρ	α_1	α_2	0	0
13	$T_{2S13} = \bar{y}^{SRS}\left(\frac{\bar{x}^{*SRS}+\rho}{\bar{X}^{SRS}+\rho}\right)^{\delta_1}\left(\frac{\bar{Z}^{SRS}+\rho}{\bar{z}^{*SRS}+\rho}\right)^{\delta_2}$	0	1	1	1	ρ	0	0	δ_1	δ_2
14	$T_{2S14} = \bar{y}^{SRS}\left(\frac{a\bar{x}^{*SRS}+\rho}{a\bar{X}^{SRS}+\rho}\right)\left(\frac{b\bar{Z}^{SRS}+\rho}{b\bar{z}^{*SRS}+\rho}\right)$	0	1	a	b	ρ	0	0	1	1
15	$T_{2S15} = \bar{y}^{SRS}\left(\frac{a\bar{X}^{SRS}+\rho}{a\bar{x}^{SRS}+\rho}\right)^{\alpha_1}\left(\frac{b\bar{z}^{SRS}+\rho}{b\bar{Z}^{SRS}+\rho}\right)^{\alpha_2}$	1	0	a	b	ρ	α_1	α_2	0	0
16	$T_{2S16} = \bar{y}^{SRS}\left(\frac{a\bar{x}^{*SRS}+\rho}{a*\bar{X}^{SRS}+\rho}\right)^{\delta_1}\left(\frac{b\bar{Z}^{SRS}+\rho}{b\bar{z}^{*SRS}+\rho}\right)^{\delta_2}$	0	1	a	b	ρ	0	0	δ_1	δ_2

3.3 Biases, MSEs and optimal MSEs of the proposed estimators

To obtain the bias and Mean Square Error of the class of estimators T_{2Ej} we write

$$\left.\begin{aligned}
\bar{y}^{ERSS_e} &= \bar{Y}^{ERSS_e}(1+e_y)\\
\bar{x}^{ERSS_e} &= \bar{X}^{ERSS_e}(1+e_x)\\
\bar{z}^{ERSS_e} &= \bar{Z}^{ERSS_e}(1+e_z)\\
E(e_y) &= E(e_x) = E(e_z) = 0\\
E(e_y^2) &= \left(\frac{1}{2\,m}\right)\frac{Var(\bar{y}^{ERSS_e})}{(\mu_y)^2} = C_y^2\\
E(e_x^2) &= \left(\frac{1}{2m}\right)\frac{Var(\bar{x}^{ERSS_e})}{(\mu_x)^2} = C_x^2\\
E(e_z^2) &= \left(\frac{1}{2m}\right)\frac{Var(\bar{z}^{ERSS_e})}{(\mu_z)^2} = C_z^2\\
\Delta_{xy} &= (\mu_{x(i)}-\mu_x)(\mu_{y[i]}-\mu_y)\\
\Delta_{yz} &= (\mu_{y[1]}-\mu_y)(\mu_{z(i)}-\mu_z)\\
\Delta_{xz} &= (\mu_{x(1)}-\mu_x)(\mu_{z(i)}-\mu_z)\\
E(e_ye_x) = C_{xy} &= \left(\frac{1}{2m}\right)\left(\rho_{xy}\cdot\frac{\sqrt{Var(\bar{y}^{ERSS_e})}}{\mu_y}\cdot\frac{\sqrt{Var(\bar{x}^{ERSS_e})}}{\mu_x} - \sum_{i=1}^{m}\Delta_{xy}\right) = \rho_{yx}C_yC_x - \frac{\phi_1}{2m}\\
E(e_ye_z) = C_{yz} &= \left(\frac{1}{2m}\right)\left(\rho_{yz}\cdot\frac{\sqrt{Var(\bar{y}^{ERSS_e})}}{\mu_y}\cdot\frac{\sqrt{Var(\bar{z}^{ERSS_e})}}{\mu_z} - \sum_{i=1}^{m}\Delta_{yz}\right) = \rho_{yz}C_yC_z - \frac{\phi_2}{2m}\\
E(e_xe_z) == C_{xz} &= \left(\frac{1}{2m}\right)\left(\rho_{xz}\cdot\frac{\sqrt{Var(\bar{x}^{ERSS_e})}}{\mu_x}\cdot\frac{\sqrt{Var(\bar{z}^{ERSS_e})}}{\mu_z} - \sum_{i=1}^{m}\Delta_{xz}\right) = \rho_{xz}C_xC_z - \frac{\phi_3}{2m}\\
\phi_1 &= \sum_{i=1}^{m}\Delta_{xy}\\
\phi_2 &= \sum_{i=1}^{m}\Delta_{yz}\\
\phi_3 &= \sum_{i=1}^{m}\Delta_{xz}
\end{aligned}\right\} \quad (21a)$$

T_{2Ej} , $T_{2(0(m))j}$, and T_{2Sj} in equations (18), (19), and (20) can be expressed in terms of $e's$ as

$$T_{2Ej}=\bar{Y}^{ERSS_e}(1+e_y)\left[w_1(1+\lambda_a e_x)^{-\alpha_1}(1+\lambda_b e_z)^{\alpha_2} + w_2(1-g\lambda_a e_x)^{\delta_1}(1-g\lambda_b e_z)^{-\delta_2}\right] \quad (22)$$

$$T_{2(0(m))j}=\bar{Y}^{ERSS_{0(m)}}(1+e_y)\left[w_1(1+\lambda_a e_x)^{-\alpha_1}(1+\lambda_b e_z)^{\alpha_2} + w_2(1-g\lambda_a e_x)^{\delta_1}(1-g\lambda_b e_z)^{-\delta_2}\right] \quad (23)$$

$$T_{2Sj}=\bar{Y}^{SRS}(1+e_0)\left[w_1(1+\lambda_a e_1)^{-\alpha_1}(1+\lambda_b e_2)^{\alpha_2} + w_2(1-g\lambda_a e_1)^{\delta_1}(1-g\lambda_b e_2)^{-\delta_2}\right] \quad (24)$$

where $\lambda_a = \frac{a*\bar{X}_{ERSS}}{a*\bar{X}_{ERSS}+\rho}$, $\lambda_1 = \frac{\bar{X}_{ERSS}}{\bar{X}_{ERSS}+\rho}$, $\lambda_b = \frac{b*\bar{Z}_{ERSS}}{b*\bar{Z}_{ERSS}+\rho}$, $\lambda_2 = \frac{\bar{Z}_{ERSS}}{\bar{Z}_{ERSS}+\rho}$.

To validate the second order of approximation, we assume that the sample size is large enough to get $|\lambda_i e_i| < 1$ and $|g\lambda_i e_i| < 1$, so that $(1+\lambda_a e_x)^{-\alpha_1}$, $(1+\lambda_b e_z)^{\alpha_2}$, $(1-g\lambda_a e_x)^{\delta_1}$, and $(1-g\lambda_b e_z)^{-\delta_2}$ can be expanded to the second order of approximation of Taylor's series. Thus; T_{2Ej} in equation (22) can now be expanded as:

$T_{2Ej} =$

$\bar{Y}^{ERSS_e}(1+e_y)\left[w_1\left((1-\alpha_1\lambda_a e_x+\frac{\alpha_1(\alpha_1+1)}{2}\lambda_a^2 e_x^2-\cdots\right)\left((1+\alpha_2 g\lambda_b e_x+\frac{\alpha_2(\alpha_2-1)}{2}\lambda_b^2 e_z^2+\cdots\right)\right]+$

$\left[w_2\left((1+g\delta_1\lambda_a e_x+\frac{\delta_1(\delta_1-1)}{2}g^2\lambda_a^2 e_x^2+\cdots\right)\left((1-g\delta_2\lambda_b e_z-\frac{\delta_2(\alpha_2-1)}{2}g^2\lambda_b^2 e_z^2+\cdots\right)\right]$

$$T_{2Ej}=\bar{Y}^{ERSS_e}(1+e_y)\left[w_1(1+\alpha_2\lambda_b g e_z+\frac{\alpha_2(\alpha_2-1)}{2}\lambda_b^2 e_z^2-\alpha_1\lambda_a e_x-\alpha_1\alpha_2\lambda_a\lambda_b g e_x e_z+\frac{\alpha_1(\alpha_1+1)}{2}\lambda_a^2 e_x^2+\cdots\right]+$$
$$\left[w_2\left((1-\delta_2\lambda_b g e_z-\frac{\delta_2(\delta_2-1)}{2}g^2\lambda_b^2 e_z^2+g\delta_1\lambda_a e_x-\delta_1\delta_2\lambda_a\lambda_b g^2 e_y e_z+\frac{\delta_1(\delta_1-1)}{2}g^2\lambda_a^2 e_x^2+\cdots\right)\right] \quad (25)$$

Expanding the right-hand side of (25) and ignoring terms of $e's$ with exponents higher than two, gives:

$$T_{2Ei}=\bar{Y}^{ERSSe}\left[w_1(1+e_y+\alpha_2\lambda_b g e_z+\alpha_2\lambda_b g e_y e_z+\frac{\alpha_2(\alpha_2-1)}{2}\lambda_b^2 e_z^2-\alpha_1\lambda_a e_y e_x-\right.$$
$$-\alpha_1\lambda_a e_x-\alpha_1\alpha_2\lambda_a\lambda_b g e_x e_z+\frac{\alpha_1(\alpha_1+1)}{2}\lambda_a^2 e_x^2+\cdots w_2\left(1+e_y-\alpha_2\lambda_b g\, e_z-\alpha_2\lambda_b g e_y\, e_z-\right.$$
$$\left.\left.\frac{\delta_2(\delta_2+1)}{2}g^2\lambda_b^2 e_z^2+g\delta_1\lambda_a g e_y e_x+\delta_1\lambda_a g\, e_x-\delta_1\delta_2\lambda_a\lambda_b g^2 e_x\, e_z+\frac{\delta_1(\delta_1-1)}{2}g^2\lambda_a^2 e_x^2+\cdots\right)\right] \quad (26)$$

$$(T_{2Ei}-\bar{Y}^{ERSSe})=$$
$$\bar{Y}^{ERSSe}\left[w_1(1+e_y+\alpha_2\lambda_b g e_z+\alpha_2\lambda_b g e_y\, e_z+\frac{\alpha_2(\alpha_2-1)}{2}\lambda_b^2 e_z^2-\alpha_1\lambda_a e_y e_x-\alpha_1\lambda_a e_x-\alpha_1\alpha_2\lambda_a\lambda_b g e_x e_z+\right.$$
$$\frac{\alpha_1(\alpha_1+1)}{2}\lambda_a^2 e_x^2+\ w_2\left(1+e_y-\alpha_2\lambda_b g\, e_z-\alpha_2\lambda_b g e_y\, e_z-\frac{\delta_2(\delta_2+1)}{2}g^2\lambda_b^2 e_z^2+\delta_1\lambda_a g e_y e_x+\delta_1\lambda_a g\, e_x-\right.$$
$$\left.\left.\delta_1\delta_2\lambda_a\lambda_b g^2 e_x\, e_z+\frac{\delta_1(\delta_1-1)}{2}g^2\lambda_a^2 e_x^2\right)-1\right] \quad (27)$$

Taking the mathematical expectations of both sides (27) yields the Bias of the estimator T_{2Ej} to the second order of approximation as:

$B(T_{2Ei})=[E(T_{2Ei})-\bar{Y}^{ERSSe})]=$

$$\bar{Y}^{ERSSe}\left[\begin{array}{l} w_1\left(\begin{array}{c}1-\alpha_1\lambda_a\left(\frac{1}{2m}\right)\left(\rho_{xy}\cdot\frac{\sqrt{Var(\bar{y}^{ERSSe})}}{\mu_y}\cdot\frac{\sqrt{Var(\bar{x}^{ERSSe})}}{\mu_x}-\sum_{i=1}^{m}\Delta_{xy}\right)+\frac{\alpha_2(\alpha_2-1)}{2}\lambda_b^2\left(\frac{1}{2m}\right)\frac{Var(\bar{z}^{ERSSe})}{(\mu_z)^2}\\ +\alpha_2\lambda_b g\left(\frac{1}{2m}\right)\left(\rho_{yz}\cdot\frac{\sqrt{Var(\bar{y}^{ERSSe})}}{\mu_y}\cdot\frac{\sqrt{Var(\bar{z}^{ERSSe})}}{\mu_z}-\sum_{i=1}^{m}\Delta_{yz}\right)\end{array}\right.\\ \left.\alpha_1\alpha_2\lambda_a\lambda_b g\left(\frac{1}{2m}\right)\left(\rho_{xz}\cdot\frac{\sqrt{Var(\bar{x}^{ERSSe})}}{\mu_x}\cdot\frac{\sqrt{Var(\bar{z}^{ERSSe})}}{\mu_z}-\sum_{i=1}^{m}\Delta_{xz}\right)+\frac{\alpha_1(\alpha_1+1)}{2}\lambda_a^2\left(\frac{1}{2m}\right)\frac{Var(\bar{x}^{ERSSe})}{(\mu_x)^2}\right)\\ +w_2\left(\begin{array}{c}1+g\delta_1\lambda_a g\left(\frac{1}{2m}\right)\left(\rho_{xy}\cdot\frac{\sqrt{Var(\bar{y}^{ERSSe})}}{\mu_y}\cdot\frac{\sqrt{Var(\bar{x}^{ERSSe})}}{\mu_x}-\sum_{i=1}^{m}\Delta_{xy}\right)-\frac{\delta_2(\delta_2+1)}{2}g^2\lambda_b^2\left(\frac{1}{2m}\right)\frac{Var(\bar{z}^{ERSSe})}{(\mu_z)^2}\\ -\alpha_2\lambda_b g\left(\frac{1}{2m}\right)\left(\rho_{yz}\cdot\frac{\sqrt{Var(\bar{y}^{ERSSe})}}{\mu_y}\cdot\frac{\sqrt{Var(\bar{z}^{ERSSe})}}{\mu_z}-\sum_{i=1}^{m}\Delta_{yz}\right)\\ -\delta_1\delta_2\lambda_a\lambda_b g^2\left(\frac{1}{2m}\right)\left(\rho_{xz}\cdot\frac{\sqrt{Var(\bar{x}^{ERSSe})}}{\mu_x}\cdot\frac{\sqrt{Var(\bar{z}^{ERSSe})}}{\mu_z}-\sum_{i=1}^{m}\Delta_{xz}\right)+\frac{\delta_1(\delta_1-1)}{2}g^2\lambda_a^2\left(\frac{1}{2m}\right)\frac{Var(\bar{x}^{ERSSe})}{(\mu_x)^2}\end{array}\right)-1\end{array}\right]$$

$$[E(T_{2Ei}) - \bar{Y}^{ERSSe})] = \frac{\bar{Y}^{ERSSe}}{2m}\left(\begin{array}{c} w_1(2m - \alpha_1\lambda_a C_{xy} + \frac{\alpha_2(\alpha_2-1)}{2}\lambda_b^2 C_z^2 + \alpha_2\lambda_b g C_{yz} \\ \alpha_1\alpha_2\lambda_a\lambda_b g C_{xz} + \frac{\alpha_1(\alpha_1+1)}{2}\lambda_a^2 C_x^2 \\ + w_2\left(\begin{array}{c} 2m + g\delta_1\lambda_a g C_{xy} - \frac{\delta_2(\delta_2+1)}{2} g^2\lambda_b^2 C_z^2 - \alpha_2\lambda_b g C_{yz} \\ -\delta_1\delta_2\lambda_a\lambda_b g^2 C_{xz} + \frac{\delta_1(\delta_1-1)}{2} g^2\lambda_a^2 C_x^2 \end{array}\right) - 2m \end{array}\right) \quad (28)$$

By taking the Squares of both sides of (27) and ignoring terms of $e's$ with exponents higher than two gives:

$$(T_{2Ei} - \bar{Y}^{ERSSe})^2 =$$

$$(\bar{Y}^{ERSSe})^2\left(\begin{array}{c} w_1^2(1 + e_y + \alpha_2\lambda_b e_y e_z + \frac{\alpha_2(\alpha_2-1)}{2}\lambda_b^2 e_z^2 - \alpha_1\lambda_a e_y e_x - \alpha_1\alpha_2\lambda_a\lambda_b e_x e_z + \frac{\alpha_1(\alpha_1+1)}{2}\lambda_a^2 e_x^2 \\ + e_y^2 + \alpha_2\lambda_b g e_y e_z - \alpha_1\lambda_a e_y e_x + \alpha_2\lambda_b e_y e_z + \alpha_2\lambda_b e_y e_z - \alpha_1\alpha_2\lambda_a\lambda_b e_x e_z + \alpha_2\lambda_b e_y e_z \\ + \frac{\alpha_2(\alpha_2-1)}{2}\lambda_a^2 e_z^2 - \alpha_1\lambda_a e_y e_x - \alpha_1\lambda_a e_y e_x - \alpha_1\alpha_2\lambda_a\lambda_b e_x e_z + \alpha_1^2\lambda_a^2 e_x^2 - \alpha_1\alpha_2\lambda_a\lambda_b e_x e_z \end{array}\right)$$

$$+ (\bar{Y}^{ERSSe})^2 w_2^2 \Big[\Big(1 - e_y - \delta_2\lambda_b g e_y e_z - \frac{\alpha_2(\alpha_2-1)}{2}\lambda_a^2 e_z^2 + \delta_1\lambda_a g\, e_y e_x + \delta_1\lambda_a g\, e_x -$$
$$\delta_1\delta_2\lambda_a\lambda_b g^2 e_x e_z + \frac{\delta_1(\delta_1-1)}{2} g^2\lambda_a^2 e_x^2 + e_y^2 - \delta_2\lambda_b g e_y e_z + \delta_1\lambda_a g\, e_y e_x - \delta_2\lambda_b g e_y e_z + \delta_2\lambda_b g\, e_z -$$
$$\delta_1\delta_2\lambda_a\lambda_b g^2 e_x e_z - \delta_2\lambda_b g e_y e_z - \frac{\delta_2(\delta_2+1)}{2} g^2\lambda_b^2 e_z^2 + \delta_1\lambda_a g\, e_y e_x + \delta_1\lambda_a g\, e_y e_x -$$
$$\delta_1\delta_2\lambda_a\lambda_b g^2 e_x e_z + +\delta_1\lambda_a g\, e_y e_x - \delta_1\delta_2\lambda_a\lambda_b g^2 e_x e_z + \frac{\delta_1(\delta_1-1)}{2} g^2\lambda_a^2 e_x^2\Big]$$
$$+ 2(\bar{Y}^{ERSSe})^2 w_1 w_2 \Big(1 - \delta_2\lambda_b g e_y e_z - \frac{\delta_2(\delta_2+1)}{2} g^2\lambda_b^2 e_z^2 + \delta_1\lambda_a g\, e_y e_x - \delta_1\delta_2\lambda_a\lambda_b g^2 e_x e_z +$$
$$\frac{\delta_1(\delta_1-1)}{2} g^2\lambda_a^2 e_x^2 + e_y^2 - \delta_2\lambda_b g e_y e_z + \delta_1\lambda_a g\, e_y e_x - \alpha_1\lambda_a e_y e_x + \alpha_2\lambda_2 e_y e_z - \alpha_2\delta_2\lambda_b g e_y e_z +$$
$$\alpha_2\delta_1\lambda_a\lambda_b g e_x e_z + \alpha_2\lambda_b g e_y e_z + \alpha_2\lambda_2 e_y e_z - \alpha_1\lambda_a e_y e_x - \alpha_1\lambda_a e_y e_x - \alpha_1\alpha_2\lambda_a\lambda_b g e_x e_z +$$
$$\alpha_1\delta_1 g\lambda_a^2 e_x^2 - -\alpha_1\alpha_2\lambda_a\lambda_b g e_x e_z + \frac{\alpha_1(\alpha_1+1)}{2}\lambda_1^2 e_x^2\Big) - 2(\bar{Y}^{ERSSe})^2 w_1 \Big(1 + \alpha_2\lambda_b e_y e_z +$$
$$\frac{\alpha_2(\alpha_2-1)}{2}\lambda_a^2 e_z^2 - \alpha_1\lambda_a e_y e_x - \alpha_1\alpha_2\lambda_a\lambda_b e_x e_z + \frac{\alpha_1(\alpha_1+1)}{2}\lambda_a^2 e_x^2\Big)$$
$$-2(\bar{Y}^{ERSSe})^2 w_2 \Big(1 - \delta_2\lambda_b g e_y e_z - \frac{\delta_2(\delta_2+1)}{2} g^2\lambda_b^2 e_z^2 + \delta_1\lambda_a g e_y e_x - \delta_1\delta_2\lambda_a\lambda_b g^2 e_x e_z +$$
$$\frac{\delta_1(\delta_1-1)}{2} g^2\lambda_a^2 e_x^2\Big) + (\bar{Y}^{ERSSe})^2 \quad (29)$$

The MSE of T_{2Ej} is obtained by taking the mathematical expectations of both sides of (29) to the second order of approximation to yield:

$$\text{MSE}(T_{2Ej}) = E(T_{2Ej} - \bar{Y}^{ERSSe})^2 =$$

$$(\bar{Y}^{ERSSe})^2 w_1^2[1 + C_y^2 + \alpha_1(\alpha_1+1)\lambda_a^2 C_x^2 + \alpha_2(\alpha_2-1)\lambda_b^2 C_z^2 - 4\alpha_1\lambda_a e_y e_x + 4\alpha_2\lambda_b C_{yz} - 4\alpha_1\alpha_2\lambda_a\lambda_b C_{xz}]$$
$$+(\bar{Y}^{ERSS})^2 w_2^2[1 + C_y^2 + \delta_1(\delta_1-1)g^2\lambda_a^2 C_x^2 - \delta_2(\delta_2+1)g^2\lambda_b^2 C_z^2 + 4\delta_1\lambda_a g C_{xy} - 4\delta_2\lambda_b g C_{yz} - 4\delta_1\delta_2\lambda_a\lambda_b g^2 C_{xz}]$$
$$+2(\bar{Y}^{ERSSe})^2 w_1 w_2\Bigg[1 + C_y^2 + [\alpha_1(\alpha_1+1) + \delta_1(\delta_1-1)g^2 + 2\alpha_1\delta_1]\frac{\lambda_a^2 C_x^2}{2} + [\alpha_2(\alpha_2-1) - \delta_2(\delta_2+1)g^2 + 2\alpha_2\delta_2]\frac{\lambda_b^2 C_z^2}{2} - 2(\alpha_1 - \delta_1 g)\lambda_a C_{xy} + 2(\alpha_2 - \delta_2 g)\lambda_b C_{yz} - (2\alpha_1\alpha_2 + \delta_1\delta_2 g - \alpha_2\delta_1)\lambda_a\lambda_b C_{xz}\Bigg]$$
$$-2(\bar{Y}^{ERSSe})^2 w_1\left[1 + \frac{\alpha_1(\alpha_1+1)}{2}\lambda_a^2 C_x^2 + \frac{\alpha_2(\alpha_2-1)}{2}\lambda_b^2 C_z^2 - \alpha_1\lambda_a C_{xy} + \alpha_2\lambda_b C_{yz} - \alpha_1\alpha_2\lambda_a\lambda_b C_{xz}\right]$$
$$-2(\bar{Y}^{ERSSe})^2 w_2\left[1 - \frac{\delta_1(\delta_1-1)}{2}g^2\lambda_a^2 C_x^2 - \frac{\delta_2(\delta_2+1)}{2}g^2\lambda_b^2 C_z^2 + \delta_1\lambda_a g C_{xy} - \delta_2\lambda_b g C_{yz} - \delta_1\delta_2\lambda_a\lambda_b g^2 C_{xz}\right]$$
$$+(\bar{Y}^{ERSSe})^2$$

$$\text{MSE}(T_{2Ej}) = E(T_{2Ej} - \bar{Y}^{ERSSe})^2 =$$

$$(\bar{Y}^{ERSSe})^2[1 + w_1^2 D_1^2 + w_2^2 D_2^2 + 2w_1 w_2 D_3 - 2w_1 D_4 - 2w_2 D_5] \quad (30)$$

where

$$D_1 = \left[1 + \frac{1}{2m}\left(C_y^2 + \alpha_1(\alpha_1+1)\lambda_a^2 C_x^2 + \alpha_2(\alpha_2-1)\lambda_b^2 C_z^2 - 4\alpha_1\lambda_a C_{yx} + 4\alpha_2\lambda_b C_{yz} - 4\alpha_1\alpha_2\lambda_a\lambda_b C_{xz}\right)\right] \quad (31)$$

$$D_2 = \left[1 + \frac{1}{2m}\left(C_y^2 + \delta_1(\delta_1-1)g^2\lambda_a^2 C_x^2 - \delta_2(\delta_2+1)g^2\lambda_b^2 C_z^2 + 4\delta_1\lambda_a g C_{xy} - 4\delta_2\lambda_b g C_{yz} - 4\delta_1\delta_2\lambda_a\lambda_b g^2 C_{xz}\right)\right] \quad (32)$$

$$D_3 = \left[1 + \frac{1}{2m}\left(C_y^2 + [\alpha_1(\alpha_1+1) + \delta_1(\delta_1-1)g^2 + 2\alpha_1\delta_1]\frac{\lambda_a^2 C_x^2}{2} + [\alpha_2(\alpha_2-1) - \delta_2(\delta_2+1)g^2 + 2\alpha_2\delta_2]\frac{\lambda_b^2 C_z^2}{2} - 2(\alpha_1 - \delta_1 g)\lambda_a C_{xy} + 2(\alpha_2 - \delta_2 g)\lambda_b C_{yz} - (2\alpha_1\alpha_2 + \delta_1\delta_2 g - \alpha_2\delta_1)\lambda_a\lambda_b C_{xz}\right)\right] \quad (33)$$

$$D_4 = \left[1 + \frac{1}{2m}\left(\frac{\alpha_1(\alpha_1+1)}{2}\lambda_a^2 C_x^2 + \frac{\alpha_2(\alpha_2-1)}{2}\lambda_b^2 C_z^2 - \alpha_1\lambda_a C_{yx} + \alpha_2\lambda_b C_{yz} - \alpha_1\alpha_2\lambda_a\lambda_b C_{xz}\right)\right] \quad (34)$$

$$D_5 = \left[1 + \frac{1}{2m}\left(-\frac{\delta_1(\delta_1-1)}{2}g^2\lambda_a^2 C_x^2 - \frac{\delta_2(\delta_2+1)}{2}g^2\lambda_b^2 C_z^2 + \delta_1\lambda_a g C_{xy} - \delta_2\lambda_b g C_{yz} - \delta_1\delta_2\lambda_a\lambda_b g^2 C_{xz}\right)\right] \quad (35)$$

In order to derive the optimum MSE of T_{2Ej} we differentiate (30) with respect to w_1 and w_2 and equate the results to zero. Thus:

$$\frac{\partial\left[MSE(T_{2Ej})\right]}{\partial w_1} = 2w_1D_1 + 2w_2D_3 - 2D_4 = 0 \quad (36)$$

$$\frac{\partial\left[MSE(T_{2Ej})\right]}{\partial w_2} = 2w_2D_2 + 2w_1D_3 - 2D_5 = 0 \quad (37)$$

Solving (36) and (37) simultaneously we have:

$$w_1D_1 + w_1D_3 + w_2D_2 + w_2D_3 - (D_4 + D_5) = 0$$

$$w_1(D_1 + D_3) + w_2(D_2 + D_3) = (D_4 + D_5) \quad (38)$$

From (36) ,

$$w_1 = \left(\frac{D_4 - w_2D_3}{D_1}\right) \quad (39)$$

Substituting (39) into (38), we have

$$\left(\frac{D_4 - w_2D_3}{D_1}\right)(D_1 + D_3) + w_2(D_2 + D_3) = (D_4 + D_5)$$

By making w_2 subject of the expression, we have

$$w_2 = \left(\frac{D_1D_5 - D_3D_4}{D_1D_2 - D_3^2}\right) \quad (40)$$

By substituting (40) into (39) or (37), gives w_1 as

$$w_1 = \left(\frac{D_2D_4 - D_3D_5}{D_1D_2 - D_3^2}\right) \quad (41)$$

The MSE of T_{2Ej} in (30) is minimized for

$$\left.\begin{aligned} w_1 &= \left(\frac{D_2D_4 - D_3D_5}{D_1D_2 - D_3^2}\right) = w_{1E0} \\ w_2 &= \left(\frac{D_1D_5 - D_3D_4}{D_1D_2 - D_3^2}\right) = w_{2E0} \end{aligned}\right\} \quad (42)$$

By substituting (42) into (30) yields the optimum or minimum MSE of T_{2Ej} as

$$MSE(T_{2Ej})_{opt} = \bar{Y}^{2^{ERSSe}}[1 + w_{1E0}^2D_1 + w_{2E0}^2D_2 + 2w_{1E0}w_{2E0}D_3 - 2w_{1E0}D_4 - 2w_{2E0}D_5]$$

$$MSE(T_{2Ej})_{opt} = \bar{Y}^{2^{ERSSe}}\left[1 + \left(\frac{D_2D_4 - D_3D_5}{D_1D_2 - D_3^2}\right)^2 D_1 + \left(\frac{D_1D_5 - D_3D_4}{D_1D_2 - D_3^2}\right)^2 D_2 + 2\left(\frac{D_2D_4 - D_3D_5}{D_1D_2 - D_3^2}\right)\left(\frac{D_1D_5 - D_3D_4}{D_1D_2 - D_3^2}\right)D_3 \right.$$
$$\left. - 2\left(\frac{D_2D_4 - D_3D_5}{D_1D_2 - D_3^2}\right)D_4 - 2\left(\frac{D_1D_5 - D_3D_4}{D_1D_2 - D_3^2}\right)D_5\right]$$

$$MSE(T_{2Ej})_{opt} = \bar{Y}^{2^{ERSSe}}\left[1 + \frac{(2D_3D_4D_5 - D_2D_4^2 - D_1D_5^2)}{(D_1D_2 - D_3^2)}\right] \quad (43)$$

3.3.1 Bias, MSE and Optimal MSE of $T_{2(0(m))j}$

Similarly, to obtain the bias and Mean Square Error of the class of estimators $T_{2(0(m))j}$ we write

$$\left.\begin{array}{c}
\bar{y}^{ERSS_{0(m)}} = \bar{Y}^{ERSS_{0(m)}}(1+e_y) \\
\bar{x}^{ERSS_{0(m)}} = \bar{X}^{ERSS_{0(m)}}(1+e_x) \\
\bar{z}^{ERSS_{0(m)}} = \bar{Z}^{ERSS_{0(m)}}(1+e_z) \\
E(e_y) = E(e_x) = E(e_z) = 0 \\
E(e_y^2) = \theta \frac{Var\left(\bar{y}^{ERSS_{0(m)}}\right)}{(\mu_y)^2} = C_y^2 \\
E(e_x^2) = \theta \frac{Var\left(\bar{x}^{ERSS_{0(m)}}\right)}{(\mu_x)^2} = C_x^2 \\
E(e_z^2) = \theta \frac{Var\left(\bar{z}^{ERSS_{0(m)}}\right)}{(\mu_z)^2} = C_z^2 \\
\Delta_{xy} = \left(\mu_{x(i)} - \mu_x\right)\left(\mu_{y[i]} - \mu_y\right) \\
\Delta_{yz} = \left(\mu_{y[1]} - \mu_y\right)\left(\mu_{z(i)} - \mu_z\right) \\
\Delta_{xz} = \left(\mu_{x(1)} - \mu_x\right)\left(\mu_{z(i)} - \mu_z\right) \\
E(e_y e_x) = C_{xy} = \theta\left(\rho_{xy} \cdot \frac{\sqrt{Var\left(\bar{y}^{ERSS_{0(m)}}\right)}}{\mu_y} \cdot \frac{\sqrt{Var\left(\bar{x}^{ERSS_{0(m)}}\right)}}{\mu_x} - \sum_{i=1}^{m} \Delta_{xy}\right) = \rho_{yx} C_y C_x - \theta\phi_1 \\
E(e_y e_z) = C_{yz} = \theta\left(\rho_{yz} \cdot \frac{\sqrt{Var\left(\bar{y}^{ERSS_{0(m)}}\right)}}{\mu_y} \cdot \frac{\sqrt{Var\left(\bar{z}^{ERSS_{0(m)}}\right)}}{\mu_z} - \sum_{i=1}^{m} \Delta_{yz}\right) = \rho_{yz} C_y C_z - \theta\phi_2 \\
E(e_x e_z) == C_{xz} = \theta\left(\rho_{xz} \cdot \frac{\sqrt{Var\left(\bar{x}^{ERSS_{0(m)}}\right)}}{\mu_x} \cdot \frac{\sqrt{Var\left(\bar{z}^{ERSS_{0(m)}}\right)}}{\mu_z} - \sum_{i=1}^{m} \Delta_{xz}\right) = \rho_{xz} C_x C_z - \theta\phi_3 \\
\phi_1 = \sum_{i=1}^{m} \Delta_{xy} \\
\phi_2 = \sum_{i=1}^{m} \Delta_{yz} \\
\phi_3 = \sum_{i=1}^{m} \Delta_{xz} \\
\theta = \left(\frac{m-1}{2m^2}\right)
\end{array}\right\} \quad (21b)$$

In like manners, the bias, MSE, and optimal MSE of $\boldsymbol{T_{2(0(m))j}}$ the odd median case of ERSS was obtained and presented as follows:

$$B(\boldsymbol{T_{2(0(m))j}}) = E\left[\left(\boldsymbol{T_{2(0(m))j}}\right) - \bar{Y}^{ERSS_{0(m)}}\right] =$$

$$\bar{Y}^{ERSS_{0(m)}}\left[\begin{array}{c} w_1\left(\begin{array}{c}\frac{1}{\theta}-\alpha_1\lambda_a\theta\left(\rho_{xy}\cdot\frac{\sqrt{Var(\bar{y}^{ERSS_{0(m)}})}}{\mu_y}\cdot\frac{\sqrt{Var(\bar{x}^{ERSS_{0(m)}})}}{\mu_x}-\sum_{i=1}^{m}\Delta_{xy}\right)+\frac{\alpha_2(\alpha_2-1)}{2}\lambda_b^2\theta\frac{Var(\bar{z}^{ERSS_{0(m)}})}{(\mu_z)^2}\\ +\alpha_2\lambda_b g\theta\left(\rho_{yz}\cdot\frac{\sqrt{Var(\bar{y}^{ERSS_{0(m)}})}}{\mu_y}\cdot\frac{\sqrt{Var(\bar{z}^{ERSS_{0(m)}})}}{\mu_z}-\sum_{i=1}^{m}\Delta_{yz}\right)\end{array}\right)\\ \alpha_1\alpha_2\lambda_a\lambda_b g\theta\left(\rho_{xz}\cdot\frac{\sqrt{Var(\bar{x}^{ERSS_{0(m)}})}}{\mu_x}\cdot\frac{\sqrt{Var(\bar{z}^{ERSS_{0(m)}})}}{\mu_z}-\sum_{i=1}^{m}\Delta_{xz}\right)+\frac{\alpha_1(\alpha_1+1)}{2}\lambda_a^2\theta\frac{Var(\bar{x}^{ERSS_{0(m)}})}{(\mu_x)^2}\\ +w_2\left(\begin{array}{c}\frac{1}{\theta}+g\delta_1\lambda_a g\theta\left(\rho_{xy}\cdot\frac{\sqrt{Var(\bar{y}^{ERSS_{0(m)}})}}{\mu_y}\cdot\frac{\sqrt{Var(\bar{x}^{ERSS_{0(m)}})}}{\mu_x}-\sum_{i=1}^{m}\Delta_{xy}\right)-\frac{\delta_2(\delta_2+1)}{2}g^2\lambda_b^2\theta\frac{Var(\bar{z}^{ERSS_{0(m)}})}{(\mu_z)^2}\\ -\alpha_2\lambda_b g\theta\left(\rho_{yz}\cdot\frac{\sqrt{Var(\bar{y}^{ERSS_{0(m)}})}}{\mu_y}\cdot\frac{\sqrt{Var(\bar{z}^{ERSS_{0(m)}})}}{\mu_z}-\sum_{i=1}^{m}\Delta_{yz}\right)\\ -\delta_1\delta_2\lambda_a\lambda_b g^2\theta\left(\rho_{xz}\cdot\frac{\sqrt{Var(\bar{x}^{ERSS_{0(m)}})}}{\mu_x}\cdot\frac{\sqrt{Var(\bar{z}^{ERSS_{0(m)}})}}{\mu_z}-\sum_{i=1}^{m}\Delta_{xz}\right)+\frac{\delta_1(\delta_1-1)}{2}g^2\lambda_a^2\theta\frac{Var(\bar{x}^{ERSS_{0(m)}})}{(\mu_x)^2}\end{array}\right)-\frac{1}{\theta}\end{array}\right]$$

$$\boldsymbol{B}(\boldsymbol{T}_{\mathbf{2(0(m))}j}) = E\left[\left(\boldsymbol{T}_{\mathbf{2(0(m))}j}\right)-\bar{Y}^{ERSS_{0(m)}}\right]=$$

$$\theta\bar{Y}^{ERSS_{0(m)}}\left(\begin{array}{c} w_1(\frac{1}{\theta}-\alpha_1\lambda_a C_{xy}+\frac{\alpha_2(\alpha_2-1)}{2}\lambda_b^2C_z^2+\alpha_2\lambda_b gC_{yz}\\ \alpha_1\alpha_2\lambda_a\lambda_b gC_{xz}+\frac{\alpha_1(\alpha_1+1)}{2}\lambda_a^2C_x^2\\ +w_2\left(\begin{array}{c}\frac{1}{\theta}+g\delta_1\lambda_a gC_{xy}-\frac{\delta_2(\delta_2+1)}{2}g^2\lambda_b^2C_z^2-\alpha_2\lambda_b gC_{yz}\\ -\delta_1\delta_2\lambda_a\lambda_b g^2C_{xz}+\frac{\delta_1(\delta_1-1)}{2}g^2\lambda_a^2C_x^2\end{array}\right)-\frac{1}{\theta}\end{array}\right) \quad (44)$$

$$MSE\left(T_{2(0(m))j}\right) = E\left(\boldsymbol{T}_{\mathbf{2(0(m))}j}-\bar{Y}^{ERSS_{0(m)}}\right)^2 =$$

$$\left(\bar{Y}^{ERSS_{0(m)}}\right)^2[1+w_1^2J_1^2+w_2^2J_2^2+2w_1w_2J_3-2w_1J_4-2w_2J_5] \quad (45)$$

where

$$J_1 = [1+\theta\left(C_y^2+\alpha_1(\alpha_1+1)\lambda_a^2C_x^2+\alpha_2(\alpha_2-1)\lambda_b^2C_z^2-4\alpha_1\lambda_aC_{yx}+4\alpha_2\lambda_b C_{yz}-4\alpha_1\alpha_2\lambda_a\lambda_bC_{xz}\right)] \quad (46)$$

$$J_2 = \left[1+\theta\left(C_y^2+\delta_1(\delta_1-1)g^2\lambda_a^2C_x^2-\delta_2(\delta_2+1)g^2\lambda_b^2C_z^2+4\delta_1\lambda_a gC_{xy}-4\delta_2\lambda_b gC_{yz}-4\delta_1\delta_2\lambda_a\lambda_b g^2C_{xz}\right)\right] \quad (47)$$

$$J_3 = \Big[1 + \theta\Big(C_y^2 + [\alpha_1(\alpha_1+1) + \delta_1(\delta_1-1)g^2 + 2\alpha_1\delta_1]\frac{\lambda_a^2 C_x^2}{2} + [\alpha_2(\alpha_2-1) - \delta_2(\delta_2+1)g^2 + 2\alpha_2\delta_2]\frac{\lambda_b^2 C_z^2}{2} - 2(\alpha_1 - \delta_1 g)\lambda_a C_{xy} + 2(\alpha_2 - \delta_2 g)\lambda_b C_{yz} - (2\alpha_1\alpha_2 + \delta_1\delta_2 g - \alpha_2\delta_1)\lambda_a\lambda_b C_{xz}\Big)\Big] \quad (48)$$

$$J_4 = \Big[1 + \theta\Big(\frac{\alpha_1(\alpha_1+1)}{2}\lambda_a^2 C_x^2 + \frac{\alpha_2(\alpha_2-1)}{2}\lambda_b^2 C_z^2 - \alpha_1\lambda_a C_{yx} + \alpha_2\lambda_b C_{yz} - \alpha_1\alpha_2\lambda_a\lambda_b C_{xz}\Big)\Big] \quad (165)$$

$$J_5 = \Big[1 - \theta\Big(\frac{\delta_1(\delta_1-1)}{2}g^2\lambda_a^2 C_x^2 - \frac{\delta_2(\delta_2+1)}{2}g^2\lambda_b^2 C_z^2 + \delta_1\lambda_a g C_{xy} - \delta_2\lambda_b g C_{yz} - \delta_1\delta_2\lambda_a\lambda_b g^2 C_{xz}\Big)\Big] \quad (49)$$

$$MSE\left(T_{2(0(m))j}\right)_{opt} = (\bar{Y}^{ERSS_{0(m)}})^2\left[1 + \frac{(2J_3J_4J_5 - J_2J_4^2 - J_1J_5^2)}{(J_1J_2 - J_3^2)}\right] \quad (50)$$

3.3.2 Bias, MSE and Optimal MSE of T_{2Sj}

Also, to obtain the bias and Mean Square Error of the class of estimators T_{2Sj} we write

$$\left.\begin{aligned}
\bar{y}^{SRS} &= \bar{Y}^{SRS}(1+e_0)\\
\bar{x}^{SRS} &= \bar{X}^{SRS}(1+e_1)\\
\bar{z}^{SRS} &= \bar{Z}^{SRS}(1+e_2)\\
E(e_0) &= E(e_1) = E(e_2) = 0\\
E(e_y^2) &= \left(\frac{1-f}{m}\right)\frac{Var(\bar{y}^{SRS})}{(\mu_y)^2} = C_0^2\\
E(e_x^2) &= \left(\frac{1-f}{m}\right)\frac{Var(\bar{x}^{SRS})}{(\mu_x)^2} = C_1^2\\
E(e_z^2) &= \left(\frac{1-f}{m}\right)\frac{Var(\bar{z}^{SRS})}{(\mu_z)^2} = C_2^2\\
E(e_0e_1) = C_{01} &= \left(\frac{1-f}{m}\right)\rho_{01}\cdot\frac{\sqrt{Var(\bar{y}^{SRS})}}{\mu_y}\cdot\frac{\sqrt{Var(\bar{x}^{SRS})}}{\mu_x} = \rho_{01}C_0C_1\\
E(e_0e_2) = C_{02} &= \left(\frac{1-f}{m}\right)\rho_{02}\cdot\frac{\sqrt{Var(\bar{y}^{SRS})}}{\mu_y}\cdot\frac{\sqrt{Var(\bar{z}^{SRS})}}{\mu_z} = \rho_{02}C_0C_2\\
E(e_1e_2) == C_{12} &= \left(\frac{m}{1-f}\right)\rho_{12}\cdot\frac{\sqrt{Var(\bar{x}^{SRS})}}{\mu_x}\cdot\frac{\sqrt{Var(\bar{z}^{SRS})}}{\mu_z} = \rho_{12}C_1C_2
\end{aligned}\right\} \quad (21c)$$

Similarly, the bias, MSE, and optimal MSE of $\boldsymbol{T_{2Sj}}$ the case of SRS was obtained and presented as follows:

$$\boldsymbol{B}(T_{2Sj}) = \left[E(T_{2Sj}) - \bar{Y}^{SRS}\right] =$$

$$\left(\frac{1-f}{m}\right)\bar{Y}^{SRS}\left(\begin{array}{c} w_1\left(\left(\frac{m}{1-f}\right) - \alpha_1\lambda_a C_{01} + \frac{\alpha_2(\alpha_2-1)}{2}\lambda_b^2 C_2^2 + \alpha_2\lambda_b g C_{02}\right.\\ \alpha_1\alpha_2\lambda_a\lambda_b g C_{12} + \frac{\alpha_1(\alpha_1+1)}{2}\lambda_a^2 C_1^2\\ + w_2\left(\begin{array}{c}\left(\frac{m}{1-f}\right) + g\delta_1\lambda_a g C_{021} - \frac{\delta_2(\delta_2+1)}{2}g^2\lambda_b^2 C_z^2 - \alpha_2\lambda_b g C_{02}\\ -\delta_1\delta_2\lambda_a\lambda_b g^2 C_{12} + \frac{\delta_1(\delta_1-1)}{2}g^2\lambda_a^2 C_1^2\end{array}\right) - \left(\frac{m}{1-f}\right)\end{array}\right) \quad (51)$$

$$MSE\left(T_{2Sj}\right) = E\left(T_{2Sj} - \overline{Y}^{SRS}\right)^2 =$$

$$\left(\overline{Y}^{SRS}\right)^2 [1 + w_1^2 L_1^2 + w_2^2 L_2^2 + 2w_1 w_2 L_3 - 2w_1 L_4 - 2w_2 L_5] \quad (52)$$

where

$$L_1 = \left[1 + \left(\frac{1-f}{m}\right)\left(C_0^2 + \alpha_1(\alpha_1+1)\lambda_a^2 C_1^2 + \alpha_2(\alpha_2-1)\lambda_b^2 C_2^2 - 4\alpha_1\lambda_a C_{yx} + 4\alpha_2\lambda_b C_{20} - 4\alpha_1\alpha_2\lambda_a\lambda_b C_{21}\right)\right] \quad (53)$$

$$L_2 = \left[1 + \left(\frac{1-f}{m}\right)\left(C_0^2 + \delta_1(\delta_1-1)g^2\lambda_a^2 C_1^2 - \delta_2(\delta_2+1)g^2\lambda_b^2 C_2^2 + 4\delta_1\lambda_a g C_{10} - 4\delta_2\lambda_b g C_{20} - 4\delta_1\delta_2\lambda_a\lambda_b g^2 C_{21}\right)\right] \quad (54)$$

$$L_3 = \left[1 + \left(\frac{1-f}{m}\right)\left(C_0^2 + [\alpha_1(\alpha_1+1) + \delta_1(\delta_1-1)g^2 + 2\alpha_1\delta_1]\frac{\lambda_a^2 C_1^2}{2} + [\alpha_2(\alpha_2-1) - \delta_2(\delta_2+1)g^2 + 2\alpha_2\delta_2]\frac{\lambda_b^2 C_2^2}{2} - 2(\alpha_1 - \delta_1 g)\lambda_a C_{10} + 2(\alpha_2 - \delta_2 g)\lambda_b C_{20} - (2\alpha_1\alpha_2 + \delta_1\delta_2 g - \alpha_2\delta_1)\lambda_a\lambda_b C_{21}\right)\right] \quad (55)$$

$$L_4 = \left[1 + \left(\frac{1-f}{m}\right)\left(\frac{\alpha_1(\alpha_1+1)}{2}\lambda_a^2 C_1^2 + \frac{\alpha_2(\alpha_2-1)}{2}\lambda_b^2 C_2^2 - \alpha_1\lambda_a C_{10} + \alpha_2\lambda_b C_{20} - \alpha_1\alpha_2\lambda_a\lambda_b C_{21}\right)\right] \quad (56)$$

$$L_5 = \left[1 + \left(\frac{1-f}{m}\right)\left(-\frac{\delta_1(\delta_1-1)}{2}g^2\lambda_a^2 C_1^2 - \frac{\delta_2(\delta_2+1)}{2}g^2\lambda_b^2 C_1^2 + \delta_1\lambda_a g C_{10} - \delta_2\lambda_b g C_{20} - \delta_1\delta_2\lambda_a\lambda_b g^2 C_{21}\right)\right] \quad (57)$$

$$MSE(T_{2Sj})_{opt} = \overline{Y}^{2\,SRS}\left[1 + \frac{\left(2L_3L_4L_5 - L_2L_4^2 - L_1L_5^2\right)}{\left(L_1L_2 - L_3^2\right)}\right] \quad (58)$$

TABLE 4
Members of $T_{2(0(m))j}$, $j = 1, 2, \dots 16$ with their MSE

S/N	$T_{2(0(m))j}$	MSE
1	$T_{2(0(m))1}$	$1/_{2m}(\bar{Y}^{ERSSe})^2(C_y^2)$
2	$T_{2(0(m))2}$	$1/_{2m}(\bar{Y}^{ERSSe})^2(C_y^2 + C_x^2 - 2C_{xy})$
3	$T_{2(0(m))3}$	$1/_{2m}(\bar{Y}^{ERSSe})^2(C_y^2 + C_z^2 + 2C_{yz})$
4	$T_{2(0(m))4}$	$1/_{2m}(\bar{Y}^{ERSSe})^2(C_y^2 + g^2C_x^2 + 2gC_{xy})$
5	$T_{2(0(m))5}$	$1/_{2m}(\bar{Y}^{ERSSe})^2(C_y^2 + g^2C_z^2 - 2gC_{yz})$
6	$T_{2(0(m))6}$	$1/_{2m}(\bar{Y}^{ERSSe})^2(C_y^2 + 3C_x^2 + C_z^2 - 2C_{xy} + 2C_{yz} - 2C_{xz})$
7	$T_{2(0(m))7}$	$1/_{2m}(\bar{Y}^{ERSSe})^2(C_y^2 + 3g^2C_z^2 + g^2C_x^2 - 4gC_{xy} - 4gC_{yz} + 4g^2C_{xz})$
8	$T_{2(0(m))8}$	$1/_{2m}(\bar{Y}^{ERSSe})^2\left(C_y^2 + 3\frac{(\alpha_1(\alpha_1+1)}{2}C_x^2 + \frac{\alpha_2(\alpha_2-1)}{2}C_z^2 - 2\alpha_1C_{xy} + 2\alpha_2C_{yz} - 2\alpha_1\alpha_2C_{xz}\right)$
9	$T_{2(0(m))9}$	$1/_{2m}(\bar{Y}^{ERSSe})^2\left(C_y^2 + 3\frac{(\delta_1(\delta_1+1)}{2}g^2C_z^2 + \frac{\delta_2(\delta_2-1)}{2}g^2C_x^2 - 4g\delta_1C_{xy} - 4g\delta_2C_{yz} + 4g^2\delta_1\delta_2C_{xz}\right)$
10	$T_{2(0(m))10}$	$1/_{2m}(\bar{Y}^{ERSSe})^2(C_y^2 + 3\lambda_1^2C_x^2 + \lambda_2^2C_z^2 - 2\lambda_1C_{xy} + 2\lambda_2C_{yz} - 2\lambda_1\lambda_2C_{xz})$
11	$T_{2(0(m))11}$	$1/_{2m}(\bar{Y}^{ERSSe})^2(C_y^2 + 3\lambda_1^2g^2C_z^2 + g^2\lambda_2^2C_x^2 - 4g\lambda_1C_{xy} - 4g\lambda_2C_{yz} + 4g^2\lambda_1\lambda_2C_{xz})$
12	$T_{2(0(m))12}$	$1/_{2m}(\bar{Y}^{ERSSe})^2\left(C_y^2 + 3\frac{(\alpha_1(\alpha_1+1)}{2}\lambda_1^2C_x^2 + \frac{\alpha_2(\alpha_2-1)}{2}\lambda_2^2C_z^2 - 2\alpha_1\lambda_1C_{xy} + 2\alpha_2\lambda_2C_{yz} - 2\alpha_1\alpha_2\lambda_1\lambda_2C_{xz}\right)$
13	$T_{2(0(m))13}$	$1/_{2m}(\bar{Y}^{ERSSe})^2\left(C_y^2 + \frac{(\delta_1(\delta_1+1)}{2}3\lambda_1^2g^2C_x^2 + \frac{\delta_2(\delta_2-1)}{2}g^2\lambda_2^2C_z^2 - 4g\lambda_1\delta_1C_{xy} - 4g\lambda_2\delta_2C_{yz} + 4g^2\delta_1\delta_2\lambda_1\lambda_2C_{xz}\right)$
14	$T_{2(0(m))14}$	$1/_{2m}(\bar{Y}^{ERSSe})^2(C_y^2 + 3\lambda_a^2g^2C_z^2 + g^2\lambda_b^2C_x^2 - 4g\lambda_aC_{xy} - 4g\lambda_bC_{yz} + 4g^2\lambda_a\lambda_bC_{xz})$
15	$T_{2(0(m))15}$	$1/_{2m}(\bar{Y}^{ERSSe})^2\left(C_y^2 + 3\frac{(\alpha_1(\alpha_1+1)}{2}\lambda_a^2C_x^2 + \frac{\alpha_2(\alpha_2-1)}{2}\lambda_a^2C_z^2 - 2\alpha_1\lambda_aC_{xy} + 2\alpha_2\lambda_aC_{yz} - 2\alpha_1\alpha_2\lambda_a\lambda_bC_{xz}\right)$
16	$T_{2(0(m))16}$	$1/_{2m}(\bar{Y}^{ERSSe})^2\left(C_y^2 + \frac{(\delta_1(\delta_1+1)}{2}3\lambda_a^2g^2C_x^2 + \frac{\delta_2(\delta_2-1)}{2}g^2\lambda_b^2C_z^2 - 4g\lambda_a\delta_1C_{xy} - 4g\lambda_b\delta_2C_{yz} + 4g^2\delta_1\delta_2\lambda_a\lambda_bC_{xz}\right)$

TABLE 5
Members of $T_{2(0(m))j}$, $j = 1, 2, \ldots 16$ with their MSE

S/N	$T_{2(0(m))j}$	MSE
1	$T_{2(0(m))1}$	$\theta(\bar{Y}^{ERSS_{0(m)}})^2(C_y^2)$
2	$T_{2(0(m))2}$	$\theta(\bar{Y}^{ERSS_{0(m)}})^2(C_y^2 + C_x^2 - 2C_{xy})$
3	$T_{2(0(m))3}$	$\theta(\bar{Y}^{ERSS_{0(m)}})^2(C_y^2 + C_z^2 + 2C_{yz})$
4	$T_{2(0(m))4}$	$\theta(\bar{Y}^{ERSS_{0(m)}})^2(C_y^2 + g^2C_x^2 + 2gC_{xy})$
5	$T_{2(0(m))5}$	$\theta(\bar{Y}^{ERSS_{0(m)}})^2(C_y^2 + g^2C_z^2 - 2gC_{yz})$
6	$T_{2(0(m))6}$	$\theta(\bar{Y}^{ERSS_{0(m)}})^2(C_y^2 + 3C_x^2 + C_z^2 - 2C_{xy} + 2C_{yz} - 2C_{xz})$
7	$T_{2(0(m))7}$	$\theta(\bar{Y}^{ERSS_{0(m)}})^2(C_y^2 + 3g^2C_z^2 + g^2C_x^2 - 4gC_{xy} - 4gC_{yz} + 4g^2C_{xz})$
8	$T_{2(0(m))8}$	$\theta(\bar{Y}^{ERSS_{0(m)}})^2\left(C_y^2 + 3\frac{(\alpha_1(\alpha_1+1)}{2}C_x^2 + \frac{\alpha_2(\alpha_2-1)}{2}C_z^2 - 2\alpha_1C_{xy} + 2\alpha_2C_{yz} - 2\alpha_1\alpha_2C_{xz}\right)$
9	$T_{2(0(m))9}$	$\theta(\bar{Y}^{ERSS_{0(m)}})^2\left(C_y^2 + 3\frac{(\delta_1(\delta_1+1)}{2}g^2C_z^2 + \frac{\delta_2(\delta_2-1)}{2}g^2C_x^2 - 4g\delta_1C_{xy} - 4g\delta_2C_{yz} + 4g^2\delta_1\delta_2C_{xz}\right)$
10	$T_{2(0(m))10}$	$\theta(\bar{Y}^{ERSS_{0(m)}})^2(C_y^2 + 3\lambda_1^2C_x^2 + \lambda_2^2C_z^2 - 2\lambda_1C_{xy} + 2\lambda_2C_{yz} - 2\lambda_1\lambda_2C_{xz})$
11	$T_{2(0(m))11}$	$\theta(\bar{Y}^{ERSS_{0(m)}})^2(C_y^2 + 3\lambda_1^2g^2C_z^2 + g^2\lambda_2^2C_x^2 - 4g\lambda_1C_{xy} - 4g\lambda_2C_{yz} + 4g^2\lambda_1\lambda_2C_{xz})$
12	$T_{2(0(m))12}$	$\theta(\bar{Y}^{ERSS_{0(m)}})^2\left(C_y^2 + 3\frac{(\alpha_1(\alpha_1+1)}{2}\lambda_1^2C_x^2 + \frac{\alpha_2(\alpha_2-1)}{2}\lambda_2^2C_z^2 - 2\alpha_1\lambda_1C_{xy} + 2\alpha_2\lambda_2C_{yz} - 2\alpha_1\alpha_2\lambda_1\lambda_2C_{xz}\right)$
13	$T_{2(0(m))13}$	$\theta(\bar{Y}^{ERSS_{0(m)}})^2\left(C_y^2 + \frac{(\delta_1(\delta_1+1)}{2}3\lambda_1^2g^2C_x^2 + \frac{\delta_2(\delta_2-1)}{2}g^2\lambda_2^2C_z^2 - 4g\lambda_1\delta_1C_{xy} - 4g\lambda_2\delta_2C_{yz} + 4g^2\delta_1\delta_2\lambda_1\lambda_2C_{xz}\right)$
14	$T_{2(0(m))14}$	$\theta(\bar{Y}^{ERSS_{0(m)}})^2(C_y^2 + 3\lambda_a^2g^2C_z^2 + g^2\lambda_b^2C_x^2 - 4g\lambda_aC_{xy} - 4g\lambda_bC_{yz} + 4g^2\lambda_a\lambda_bC_{xz})$
15	$T_{2(0(m))15}$	$\theta(\bar{Y}^{ERSS_{0(m)}})^2\left(C_y^2 + 3\frac{(\alpha_1(\alpha_1+1)}{2}\lambda_a^2C_x^2 + \frac{\alpha_2(\alpha_2-1)}{2}\lambda_a^2C_z^2 - 2\alpha_1\lambda_aC_{xy} + 2\alpha_2\lambda_aC_{yz} - 2\alpha_1\alpha_2\lambda_a\lambda_bC_{xz}\right)$
16	$T_{2(0(m))16}$	$\theta(\bar{Y}^{ERSS_{0(m)}})^2\left(C_y^2 + \frac{(\delta_1(\delta_1+1)}{2}3\lambda_a^2g^2C_x^2 + \frac{\delta_2(\delta_2-1)}{2}g^2\lambda_b^2C_z^2 - 4g\lambda_a\delta_1C_{xy} - 4g\lambda_b\delta_2C_{yz} + 4g^2\delta_1\delta_2\lambda_a\lambda_bC_{xz}\right)$

TABLE 6
Members of T_{2Sj} , $j = 1, 2, \dots 16$ with their MSE

S/N	T_{2Sj}	MSE
1	T_{2S1}	$\left(\frac{1-f}{m}\right)(\bar{Y}^{SRS})^2(C_0^2)$
2	T_{2S2}	$\left(\frac{1-f}{m}\right)(\bar{Y}^{SRS})^2(C_0^2 + C_1^2 - 2C_{10})$
3	T_{2S3}	$\left(\frac{1-f}{m}\right)(\bar{Y}^{SRS})^2(C_0^2 + C_2^2 + 2C_{20})$
4	T_{2S4}	$\left(\frac{1-f}{m}\right)(\bar{Y}^{SRS})^2(C_0^2 + g^2C_1^2 + 2gC_{10})$
5	T_{2S5}	$\left(\frac{1-f}{m}\right)(\bar{Y}^{SRS})^2(C_0^2 + g^2C_2^2 - 2gC_{20})$
6	T_{2S6}	$\left(\frac{1-f}{m}\right)(\bar{Y}^{SRS})^2(C_0^2 + 3C_1^2 + C_2^2 - 2C_{10} + 2C_{20} - 2C_{21})$
7	T_{2S7}	$\left(\frac{1-f}{m}\right)(\bar{Y}^{SRS})^2\left(C_0^2 + g^2C_2^2 + 3g^2C_1^2 - 4gC_{10} - 4gC_{yz} + 4g^2C_{xz}\right)$
8	T_{2S8}	$\left(\frac{1-f}{m}\right)(\bar{Y}^{SRS})^2\left(C_0^2 + 3\frac{(\alpha_1(\alpha_1+1)}{2}C_1^2 + \frac{\alpha_2(\alpha_2-1)}{2}C_2^2 - 2\alpha_1C_{10} + 2\alpha_2C_{20} - 2\alpha_1\alpha_2C_{21}\right)$
9	T_{2S9}	$\left(\frac{1-f}{m}\right)(\bar{Y}^{SRS})^2\left(C_0^2 + 3\frac{(\delta_1(\delta_1+1)}{2}g^2C_1^2 + \frac{\delta_2(\delta_2-1)}{2}g^2C_2^2 - 4g\delta_1C_{10} - 4g\delta_2C_{20} + 4g^2\delta_1\delta_2C_{21}\right)$
10	T_{2S10}	$\left(\frac{1-f}{m}\right)(\bar{Y}^{SRS})^2(C_0^2 + 3\lambda_1^2C_1^2 + \lambda_2^2C_2^2 - 2\lambda_1C_{10} + 2\lambda_2C_{20} - 2\lambda_1\lambda_2C_{21})$
11	T_{2S11}	$\left(\frac{1-f}{m}\right)(\bar{Y}^{SRS})^2(C_0^2 + g^2\lambda_2^2C_2^2 + 3\lambda_1^2g^2C_1^2 - 4g\lambda_1C_{10} - 4g\lambda_2C_{21} + 4g^2\lambda_1\lambda_2C_{21})$
12	T_{2S12}	$\left(\frac{1-f}{m}\right)(\bar{Y}^{SRS})^2\left(C_0^2 + 3\frac{(\alpha_1(\alpha_1+1)}{2}\lambda_1^2C_1^2 + \frac{\alpha_2(\alpha_2-1)}{2}\lambda_2^2C_2^2 - 2\alpha_1\lambda_1C_{10} + 2\alpha_2\lambda_2C_{20} - 2\alpha_1\alpha_2\lambda_1\lambda_2C_{21}\right)$
13	T_{2S13}	$\left(\frac{1-f}{m}\right)(\bar{Y}^{SRS})^2\left(C_0^2 + \frac{(\delta_1(\delta_1+1)}{2}3\lambda_1^2g^2C_1^2 + \frac{\delta_2(\delta_2-1)}{2}g^2\lambda_2^2C_2^2 - 4g\lambda_1\delta_1C_{10} - 4g\lambda_2\delta_2C_{20} + 4g^2\delta_1\delta_2\lambda_1\lambda_2C_{21}\right)$
14	T_{2S14}	$\left(\frac{1-f}{m}\right)(\bar{Y}^{SRS})^2(C_0^2 + g^2\lambda_b^2C_2^2 + 3\lambda_a^2g^2C_1^2 - 4g\lambda_aC_{10} - 4g\lambda_bC_{20} + 4g^2\lambda_a\lambda_bC_{21})$
15	T_{2S15}	$\left(\frac{1-f}{m}\right)(\bar{Y}^{SRS})^2\left(C_0^2 + 3\frac{(\alpha_1(\alpha_1+1)}{2}\lambda_a^2C_1^2 + \frac{\alpha_2(\alpha_2-1)}{2}\lambda_a^2C_2^2 - 2\alpha_1\lambda_aC_{10} + 2\alpha_2\lambda_aC_{20} - 2\alpha_1\alpha_2\lambda_a\lambda_bC_{21}\right)$
16	T_{2S16}	$\left(\frac{1-f}{m}\right)(\bar{Y}^{SRS})^2\left(C_0^2 + 3\frac{(\delta_1(\delta_1+1)}{2}\lambda_a^2g^2C_1^2 + \frac{\delta_2(\delta_2-1)}{2}g^2\lambda_b^2C_2^2 - 4g\lambda_a\delta_1C_{10} - 4g\lambda_b\delta_2C_{20} + 4g^2\delta_1\delta_2\lambda_a\lambda_bC_{21}\right)$

3.4 Efficiency comparison

Let $MSE(T_{2Ej})_{opt}$, $MSE(T_{2[0(m)]j})_{opt}$, and $\text{MSE}(T_{2Sj})_{opt}$ be the Mean Square Errors (MSEs) of T_{2Ej} , $T_{2[0(m)]j}$, T_{2Sj}, under ERSS for $ERSSe$ case, e: is even, and $ERSS_{0(m)}$ case, $0(m)$ is median odd, and that of the estimator proposed under SRS respectively.

(i) $MSE\left(T_{2Ej}\right)_{opt}$ is more efficient than $MSE\left(T_{2Sj}\right)_{opt}$ under optimal condition if,

$$\frac{\left(\bar{Y}^{ERSS_e}\right)^2\left[1+\frac{\left(2D_3D_4D_5-D_2D_4^2-D_1D_5^2\right)}{\left(D_1D_2-D_3^2\right)}\right]}{\left(\bar{Y}^{SRS}\right)^2\left[1+\frac{\left(2L_3L_4L_5-L_2L_4^2-L_1L_5^2\right)}{\left(L_1L_2-L_3^2\right)}\right]}<1 \quad or \quad \frac{1}{\frac{\left(\bar{Y}^{ERSS_e}\right)^2\left[1+\frac{\left(2D_3D_4D_5-D_2D_4^2-D_1D_5^2\right)}{\left(D_1D_2-D_3^2\right)}\right]}{\left(\bar{Y}^{SRS}\right)^2\left[1+\frac{\left(2L_3L_4L_5-L_2L_4^2-L_1L_5^2\right)}{\left(L_1L_2-L_3^2\right)}\right]}}>1,$$

$$\text{for } j=1 \text{ to } 16 \qquad (59)$$

(ii) $MSE\left(T_{2[0(m)]j}\right)_{opt}$ is more efficient than $MSE\left(T_{2Sj}\right)_{opt}$ if,

$$\frac{\left(\bar{Y}^{ERSS_{o(m)}}\right)^2\left[1+\frac{\left(2J_3J_4J_5-J_2J_4^2-J_1J_5^2\right)}{\left(J_1J_2-J_3^2\right)}\right]}{\left(\bar{Y}^{SRS}\right)^2\left[1+\frac{\left(2L_3L_4L_5-L_2L_4^2-L_1L_5^2\right)}{\left(L_1L_2-L_3^2\right)}\right]}<1 \quad or \quad \frac{1}{\frac{\left(\bar{Y}^{ERSS_{o(m)}}\right)^2\left[1+\frac{\left(2J_3J_4J_5-J_2J_4^2-J_1J_5^2\right)}{\left(J_1J_2-J_3^2\right)}\right]}{\left(\bar{Y}^{SRS}\right)^2\left[1+\frac{\left(2L_3L_4L_5-L_2L_4^2-L_1L_5^2\right)}{\left(L_1L_2-L_3^2\right)}\right]}}>1,$$

$$\text{for } j=1 \text{ to} 16 \qquad (60)$$

(iii) $MSE\left(T_{2[0(m)]j}\right)_{opt}$, is most efficient than $MSE\left(T_{2Ej}\right)_{opt}$ and $MSE\left(T_{2Sj}\right)_{opt}$ if,

$$\left(\bar{Y}^{ERSS_{o(m)}}\right)^2\left[1+\frac{\left(2J_3J_4J_5-J_2J_4^2-J_1J_5^2\right)}{\left(J_1J_2-J_3^2\right)}\right]<\left(\bar{Y}^{ERSS_e}\right)^2\left[1+\frac{\left(2D_3D_4D_5-D_2D_4^2-D_1D_5^2\right)}{\left(D_1D_2-D_3^2\right)}\right]<$$

$$\left(\bar{Y}^{SRS}\right)^2\left[1+\frac{\left(2L_3L_4L_5-L_2L_4^2-L_1L_5^2\right)}{\left(L_1L_2-L_3^2\right)}\right] \qquad (61)$$

(iv) $MSE\ \left(T_{2Ej}\right)_{opt}$ is more efficient than $MSE\left(T_{2Sj}\right)_{opt}$ in terms of PRE, if

$$\left.\begin{array}{c}\dfrac{\left(\bar{Y}^{ERSS_{o(m)}}\right)^2\left[1+\frac{\left(2J_3J_4J_5-J_2J_4^2-J_1J_5^2\right)}{\left(J_1J_2-J_3^2\right)}\right]}{\left(\bar{Y}^{SRS}\right)^2\left[1+\frac{\left(2L_3L_4L_5-L_2L_4^2-L_1L_5^2\right)}{\left(L_1L_2-L_3^2\right)}\right]}\times 100<100, \quad or \\ \dfrac{1}{\frac{\left(\bar{Y}^{ERSS_{o(m)}}\right)^2\left[1+\frac{\left(2J_3J_4J_5-J_2J_4^2-J_1J_5^2\right)}{\left(J_1J_2-J_3^2\right)}\right]}{\left(\bar{Y}^{SRS}\right)^2\left[1+\frac{\left(2L_3L_4L_5-L_2L_4^2-L_1L_5^2\right)}{\left(L_1L_2-L_3^2\right)}\right]}}\times 100>100 \\ \text{for } j=1 \text{ to } 16\end{array}\right\} \qquad (62)$$

(v) $MSE\ \left(T_{2[0(m)]j}\right)_{opt}$ is more efficient than $MSE\left(T_{2Sj}\right)_{opt}$ in terms of PRE, if,

$$\left.\begin{array}{l} \dfrac{(\bar{Y}^{ERSS_e})^2\left[1+\dfrac{\left(2D_3D_4D_5-D_2D_4^2-D_1D_5^2\right)}{(D_1D_2-D_3^2)}\right]}{(\bar{Y}^{SRS})^2\left[1+\dfrac{(2L_3L_4L_5-L_2L_4^2-L_1L_5^2)}{(L_1L_2-L_3^2)}\right]}\times 100<100\,, \qquad or \\ \dfrac{1}{\dfrac{(\bar{Y}^{ERSS_e})^2\left[1+\dfrac{(2D_3D_4D_5-D_2D_4^2-D_1D_5^2)}{(D_1D_2-D_3^2)}\right]}{(\bar{Y}^{SRS})^2\left[1+\dfrac{(2L_3L_4L_5-L_2L_4^2-L_1L_5^2)}{(L_1L_2-L_3^2)}\right]}}\times 100>100 \\ \text{for } j=1\ to\ 16 \end{array}\right\} \qquad (63)$$

4.0 Empirical study

To investigate the efficiency of T_{2Ej} , $T_{2[0(m)]j}$ and its members under ERSS over its corresponding counterpart T_{2Sj}, based on SRS, and some existing ratio type estimators, we have considered three natural populations data sets. The real-life data sets were obtained from various sources and the description of the population and the values of the required parameters are as given below:

Population I: [Source: Singh (1969)]
Y: Number of females employed,
X: Number of females in service
Z: Number of educated females
$M = 61$, $m = 20$, $\bar{Y} = 7.46$, $\bar{X} = 5.31$, $\bar{Z} = 179.00$
$\rho_{xy} = 0.7737$, $\rho_{yz} = -0.2070$, $\rho_{xz} = -0.0033$, $C_y^2 = 0.5046$, $C_x^2 = 0.5737$, $C_z^2 = 0.0633$

Population II: [Source: Steel and Torrie (1960)]
Y: Log of leaf burn in seconds,
X: Potassium percentage
Z: Chlorine Percentage
$M = 30$, $m = 6$, $\bar{Y} = 0.6860$, $\bar{X} = 4.6437$, $\bar{Z} = 0.8077$
$\rho_{xy} = 0.1794$, $\rho_{yz} = -0.4996$, $\rho_{xz} = 0.4074$, $C_y^2 = 0.4803$, $C_x^2 = 0.2295$, $C_z^2 = 0.7493$

Population III: [Source: Khare and Rehman (2015)]
Y: Number of Agricultural labour
X: Area of village hectares
Z: Number of Cultivators in the village
$M = 96$, $m = 24$, $\bar{Y} = 137.9271$, $\bar{X} = 144.8720$, $\bar{Z} = 185.188$, $C_x = 0.8115$, $C_y = 1.3232$,
$C_z = 1.5521$ $\rho_{xy} = 0.786$, $\rho_{yz} = 0.786$, $\rho_{xz} = 0.819$

Fixed values of scalars $\alpha_1 = 0.5$, $\alpha_2 = -1$, $\delta_1 = -1$, $\delta_2 = -0.5$

and setting $\phi_1 = \phi_2 = \phi_3 = 0$

TABLE 7

Biases of the members of T_{2Ej}, $T_{2(o(m))j}$, T_{2Sj}

ESTIMATORS/POPULATIONS

	T_{2Ej}				$T_{2(o(m))j}$				T_{2Sj}		
members	*I*	*II*	*III*	*members*	*I*	*II*	*III*	*members*	*I*	*II*	*III*
T_{2E1}	0.00	0.00	0.00	$T_{2(o(m))1}$	0.00	0.00	0.00	T_{2S1}	0.00	0.00	0.00
T_{2E2}	0.0293567	0.0099838	-0.492794	$T_{2(o(m))2}$	0.0278889	0.0083198	-0.472261	T_{2S2}	0.0394631	0.0159741	-0.739191
T_{2E3}	-0.0001173	0.0096579	2.9641522	$T_{2(o(m))3}$	-0.000111	0.0080482	2.8406458	T_{2S3}	-0.0001577	0.01545258	4.4462282
T_{2E4}	-0.037871	-0.000784	-0.795925	$T_{2(o(m))4}$	-0.035977	-0.000653	-0.761899	T_{2S4}	-0.0001577	0.01545258	-1.192537
T_{2E5}	-0.0124118	-3.6E-06	-0.5847714	$T_{2(o(m))5}$	-0.035977	-0.000653	-0.761899	T_{2S5}	-0.016685	-0.0000057	-0.877157
T_{2E6}	0.022575	-0.015454	1.181538	$T_{2(o(m))6}$	0.021446	-0.012878	1.132307	T_{2S6}	0.0303461	-0.024726	1.7723074
T_{2E7}	-0.04399	-0.00982	-1.00605	$T_{2(o(m))7}$	-0.041789	-0.008184	-0.9641348	T_{2S7}	-0.059132	-0.0157128	-1.509081
T_{2E8}	0.0442068	0.0537136	1.7576993	$T_{2(o(m))8}$	0.0419965	0.447613	1.6844618	T_{2S8}	0.0594256	0.0859417	2.6365489
T_{2E9}	-0.040593	-0.004062	-0.475046	$T_{2(o(m))9}$	-0.038564	-0.003385	-0.455253	T_{2S9}	-0.054568	-0.006499	-0.71257
T_{2E10}	0.006942	-0.056602	1.182753	$T_{2(o(m))10}$	0.006595	-0.047168	1.133472	T_{2S10}	0.0093318	-0.0905626	1.7741295
T_{2E11}	-0.05577	-0.017952	-0.437859	$T_{2(o(m))11}$	-0.052981	-0.01496	-0.419615	T_{2S11}	-0.074969	-0.0287225	-0.656788
T_{2E12}	0.042483	0.323551	1.729186	$T_{2(o(m))12}$	-0.040359	0.0269626	1.657137	T_{2S12}	0.0571089	0.5176816	2.593779
T_{2E13}	-0.035783	-0.013587	-0.470095	$T_{2(o(m))13}$	-0.033994	-0.011323	-0.450508	T_{2S13}	-0.408102	-0.02174	-0.705143
T_{2E14}	0.0400154	0.0467684	-0.337717	$T_{2(o(m))14}$	0.0380146	0.0389737	-0.323645	T_{2S14}	0.0537912	0.0748295	-0.506575
T_{2E15}	-0.034503	-0.020938	-0.468869	$T_{2(o(m))15}$	-0.032778	-0.017448	-0.449332	T_{2S15}	-0.046381	-0.0335	0.703303
T_{2E16}	-0.034503	-0.020938	-0.468869	$T_{2(o(m))16}$	-0.032778	-0.017448	-0.044332	T_{2S16}	-0.046381	-0.0335	0.703303

TABLE 8

Results of Relative biases of T_{2Ej}, $T_{2(o(m))j}$, T_{2Sj}, $j=1,2..16$

ESTIMATORS/POPULATIONS

	T_{2Ej}				$T_{2(o(m))j}$				T_{2Sj}		
members	*I*	*II*	*III*	*members*	*I*	*II*	*III*	*members*	*I*	*II*	*III*
T_{2E1}	0.00	0.00	0.00	$T_{2(o(m))1}$	0.00	0.00	0.00	T_{2S1}	0.00	0.00	0.00
T_{2E2}	0.0039352	0.01455364	-0.0035729	$T_{2(o(m))2}$	0.0037385	0.01212799	-0.003424	T_{2S2}	0.00529	0.0232859	-0.005359
T_{2E3}	-1.572E-05	0.01407851	0.02149072	$T_{2(o(m))3}$	-1.493E-05	0.0117321	0.02059527	T_{2S3}	-2.114E-05	0.0225256	0.0322361
T_{2E4}	-0.0050765	-0.0011429	-0.0057706	$T_{2(o(m))4}$	-0.0048227	-0.0009519	-0.0055239	T_{2S4}	-2.114E-05	0.0225256	-0.008646
T_{2E5}	-0.0016638	-5.248E-06	-0.0042397	$T_{2(o(m))5}$	-0.0048227	-0.0009519	-0.0055239	T_{2S5}	-0.0022366	-8.309E-06	-0.00636
T_{2E6}	0.0030261	-0.0225277	0.00856639	$T_{2(o(m))6}$	0.0028748	-0.0187726	0.00820946	T_{2S6}	0.0040678	-0.0360437	0.0128496
T_{2E7}	-0.0058968	-0.0143149	-0.0072941	$T_{2(o(m))7}$	-0.0056017	-0.0119297	-0.0069902	T_{2S7}	-0.0079265	-0.022905	-0.010941
T_{2E8}	0.0059258	0.07829971	0.01274368	$T_{2(o(m))8}$	0.0056296	0.65249708	0.0122127	T_{2S8}	0.0079659	0.1252794	0.0191155
T_{2E9}	-0.0054414	-0.0059213	-0.0034442	$T_{2(o(m))9}$	-0.0051694	-0.0049344	-0.0033007	T_{2S9}	-0.0073147	-0.0094738	-0.005166
T_{2E10}	0.0009306	-0.0825102	0.0085752	$T_{2(o(m))10}$	0.000884	-0.068758	0.00821791	T_{2S10}	0.0012509	-0.1320155	0.0128628
T_{2E11}	-0.0074759	-0.0261691	-0.0031746	$T_{2(o(m))11}$	-0.007102	-0.0218076	-0.0030423	T_{2S11}	-0.0100495	-0.0418696	-0.004762
T_{2E12}	0.0056948	0.47164869	0.01253696	$T_{2(o(m))12}$	-0.0054101	0.03930408	0.01201459	T_{2S12}	0.0076553	0.7546379	0.0188054
T_{2E13}	-0.0047966	-0.0198061	-0.0034083	$T_{2(o(m))13}$	-0.0045568	-0.0165058	-0.0032663	T_{2S13}	-0.0547054	-0.031691	-0.005112
T_{2E14}	0.005364	0.06817551	-0.0024485	$T_{2(o(m))14}$	0.0050958	0.05681297	-0.0023465	T_{2S14}	0.0072106	0.1090809	-0.003673
T_{2E15}	-0.0046251	-0.0305219	-0.0033994	$T_{2(o(m))15}$	-0.0043938	-0.0254344	-0.0032577	T_{2S15}	-0.0062173	-0.0488338	0.0050991
T_{2E16}	-0.0046251	-0.0305219	-0.0033994	$T_{2(o(m))16}$	-0.0043938	-0.0254344	-0.0003214	T_{2S16}	-0.0062173	-0.0488338	0.0050991

TABLE 9

MSEs of members of T_{2Ej}, $T_{2(o(m))j}$, T_{2Sj}, *j=1,2..16*

ESTIMATORS/POPULATIONS

	T_{2Ej}				$T_{2(o(m))j}$				T_{2Sj}		
members	*I*	*II*	*III*	*members*	*I*	*II*	*III*	*members*	*I*	*II*	*III*
T_{2E1}	0.7020409	0.0159763	693.91929	$T_{2(o(m))1}$	0.6668939	0.0133136	665.00598	T_{2S1}	0.94372715	0.02556206	1040.8789
T_{2E2}	0.3418869	0.020674	296.98328	$T_{2(o(m))2}$	0.3247869	0.0172283	284.60897	T_{2S2}	0.45957765	0.00330783	445.47492
T_{2E3}	0.6871662	0.0237113	2928.2316	$T_{2(o(m))3}$	0.6528079	0.0197595	2806.222	T_{2S3}	0.92373167	0.03793815	4392.3475
T_{2E4}	1.4570018	0.0176144	942.22982	$T_{2(o(m))4}$	1.3841517	0.0146787	902.97025	T_{2S4}	1.9585926	0.028183	1413.3447
T_{2E5}	0.773211	0.0232252	373.48936	$T_{2(o(m))5}$	0.7345504	0.0193544	357.9273	T_{2S5}	1.0393983	0.0371604	560.23404
T_{2E6}	1.9250806	0.0331586	2235.6152	$T_{2(o(m))6}$	1.8288266	0.0276322	2142.4646	T_{2S6}	2.5878133	0.05305375	3353.4228
T_{2E7}	2.2324722	0.0091704	1996.9502	$T_{2(o(m))7}$	2.2070849	0.007642	1913.7439	T_{2S7}	3.12305364	0.01467265	2995.4253
T_{2E8}	1.1737712	0.0591616	191.39648	$T_{2(o(m))8}$	1.1150827	0.0493013	183.42163	T_{2S8}	1.5778564	0.0946585	287.09473
T_{2E9}	1.8776931	0.0490735	3471.3873	$T_{2(o(m))9}$	1.7838084	0.0408946	3326.7462	T_{2S9}	2.524112	0.0785176	5207.081
T_{2E10}	1.5008898	0.17699	2233.3076	$T_{2(o(m))10}$	1.4258453	0.1474992	2140.2531	T_{2S10}	2.0775895	0.2831984	3349.9614
T_{2E11}	2.2447393	0.0564308	292.07935	$T_{2(o(m))11}$	2.1325023	0.0470257	279.90938	T_{2S11}	3.01751835	0.09028931	438.11902
T_{2E12}	1.171618	0.2568978	191.92976	$T_{2(o(m))12}$	1.1130337	0.0214082	183.93269	T_{2S12}	1.57496194	0.41103649	281.89464
T_{2E13}	1.7356277	0.0996841	3444.6024	$T_{2(o(m))13}$	1.6488463	0.0830701	3301.0773	T_{2S13}	2.33313883	0.15949463	5166.9036
T_{2E14}	2.1732923	0.0745094	290.31931	$T_{2(o(m))14}$	2.0646277	0.0620911	278.22267	T_{2S14}	2.9214749	0.119215	435.47896
T_{2E15}	1.1702729	0.0430124	192.58942	$T_{2(o(m))15}$	1.1117593	0.3584368	184.56486	T_{2S15}	1.57315374	0.68819861	288.88412
T_{2E16}	1.7085193	0.04026977	653.87618	$T_{2(o(m))16}$	1.6230934	0.0355815	625.63134	T_{2S16}	2.2966982	0.06831639	980.81427
								Singh & Taylor (2005)	2.587813	0.05305375	3353.4228
								Singh et al(2011)	2.42704049	0.05423914	853.15024
								Vishwakarma & Kumar (2015)	1.43922069	0.08434752	1467.46056

TABLE 10

Results of relative efficiency of T_{2Ej}, $T_{2(o(m))j}$, T_{2Sj}, *j=1,2..16*

ESTIMATORS/POPULATIONS

	$RE(T_{2(o(m))j}, T_{2Ej})$				$RE(T_{2Ej}, T_{2Sj})$				$RE(T_{2(o(m))j}, T_{2Sj})$		
members	*I*	*II*	*III*	*members*	*I*	*II*	*III*	*members*	*I*	*II*	*III*
$(T_{2(o(m))1}, T_{2E1})$	0.9499359	0.8333346	0.9583333	(T_{2E1}, T_{2S1})	0.7439024	0.6249989	0.6666667	$(T_{2(o(m))1}, T_{2S1})$	0.70665964	0.52083322	0.6388889
$(T_{2(o(m))2}, T_{2E2})$	0.9499833	0.8333318	0.9583333	(T_{2E2}, T_{2S2})	0.7439155	6.2500125	0.6666667	$(T_{2(o(m))2}, 2_{1S2})$	0.70670732	5.2083344	0.6388889
$(T_{2(o(m))3}, T_{2E3})$	0.95	0.8333333	0.9583333	(T_{2E3}, T_{2S3})	0.7439024	0.6249999	0.6666667	$(T_{2(o(m))3}, T_{2S3})$	0.70670732	0.52083325	0.6388889
$(T_{2(o(m))4}, T_{2E4})$	0.95	0.8333352	0.9583333	(T_{2E4}, T_{2S4})	0.7439024	0.6250009	0.6666667	$(T_{2(o(m))4}, T_{2S4})$	0.70670731	0.52083526	0.6388889
$(T_{2(o(m))5}, T_{2E5})$	0.9499999	0.8333362	0.9583333	(T_{2E5}, T_{2S5})	0.7439025	0.6249987	0.6666667	$(T_{2(o(m))5}, T_{2S5})$	0.70670733	0.52083401	0.6388889
$(T_{2(o(m))6}, T_{2E6})$	0.95	0.8333334	0.9583333	(T_{2E6}, T_{2S6})	0.7439024	0.6249999	0.6666667	$(T_{2(o(m))6}, T_{2S6})$	0.70670732	0.52083331	0.6388889
$(T_{2(o(m))7}, T_{2E7})$	0.9886281	0.8333335	0.9583333	(T_{2E7}, T_{2S7})	0.7148363	0.6250003	0.6666667	$(T_{2(o(m))7}, T_{2S7})$	0.70670732	0.52083366	0.6388889
$(T_{2(o(m))8}, T_{2E8})$	0.9500001	0.8333328	0.9583334	(T_{2E8}, T_{2S8})	0.7439024	0.6250004	0.6666666	$(T_{2(o(m))8}, T_{2S8})$	0.70670734	0.52083331	0.6388889
$(T_{2(o(m))9}, T_{2E9})$	0.95	0.8333337	0.9583333	(T_{2E9}, T_{2S9})	0.7439024	0.625	0.6666667	$(T_{2(o(m))9}, T_{2S9})$	0.70670731	0.52083355	0.6388889
$(T_{2(o(m))10}, T_{2E10})$	0.95	0.8333759	0.9583333	(T_{2E10}, T_{2S10})	0.7224188	0.6249682	0.6666667	$(T_{2(o(m))10}, T_{2S10})$	0.68629789	0.52083345	0.6388889
$(T_{2(o(m))11}, T_{2E11})$	0.95	0.8333333	0.9583333	(T_{2E11}, T_{2S11})	0.7439024	0.625	0.6666667	$(T_{2(o(m))11}, T_{2S11})$	0.70670732	0.52083331	0.6388889
$(T_{2(o(m))12}, T_{2E12})$	0.9499971	0.0833333	0.9583333	(T_{2E12}, T_{2S12})	0.7439024	0.625	0.6808564	$(T_{2(o(m))12}, T_{2S12})$	0.70670515	0.05208333	0.6524874
$(T_{2(o(m))13}, T_{2E13})$	0.95	0.8333335	0.9583333	(T_{2E13}, T_{2S13})	0.7439025	0.6249997	0.6666667	$(T_{2(o(m))13}, T_{2S13})$	0.70670732	0.52083321	0.6388889
$(T_{2(o(m))14}, T_{2E14})$	0.95	0.8333324	0.9583333	(T_{2E14}, T_{2S14})	0.7439024	0.6250002	0.6666667	$(T_{2(o(m))14}, T_{2S14})$	0.70670732	0.52083295	0.6388889
$(T_{2(o(m))15}, T_{2E15})$	95	0.833334	0.9583333	(T_{2E15}, T_{2S15})	0.7439025	0.625000	0.6666667	$(T_{2(o(m))15}, T_{2S15})$	0.70670732	0.52083321	0.6388889
$(T_{2(o(m))16}, T_{2E16})$	0.950005	0.835771	0.9583333	(T_{2E16}, T_{2S16})	0.7439024	0.0624999	0.6666667	$(T_{2(o(m))16}, T_{2S16})$	0.70670732	0.52083295	0.6388889

TABLE 11

Percent relative efficiency of members of T_{2Ej}, $T_{2(o(m))j}$, T_{2Sj}, j=1,2..16

ESTIMATORS/POPULATIONS

	$PRE(T_{2(o(m))j}, T_{2Ej})$				$PRE(T_{2Ej}, T_{2Sj})$				$PRE(T_{2(o(m))j}, T_{2Sj})$		
members	*I*	*II*	*III*	*members*	*I*	*II*	*III*	*members*	*I*	*II*	*III*
$(T_{2(o(m))1}, T_{2E1})$	94.993591	83.333459	95.833333	(T_{2E1}, T_{2S1})	74.390244	62.499892	66.666667	$(T_{2(o(m))1}, T_{2S1})$	70.665964	52.0833219	63.888889
$(T_{2(o(m))2}, T_{2E2})$	94.998335	83.333184	95.833333	(T_{2E2}, T_{2S2})	74.391548	625.00125	66.666667	$(T_{2(o(m))2}, 2_{1S2})$	70.6707321	52..83344	63.888889
$(T_{2(o(m))3}, T_{2E3})$	95	83.333333	95.833333	(T_{2E3}, T_{2S3})	74.390244	62.49999	66.666667	$(T_{2(o(m))3}, T_{2S3})$	70.6707317	52.0833251	63.888889
$(T_{2(o(m))4}, T_{2E4})$	94.999999	83.333523	95.833334	(T_{2E4}, T_{2S4})	74.390243	62.500089	66.666668	$(T_{2(o(m))4}, T_{2S4})$	70.6707306	52.0835255	63.888891
$(T_{2(o(m))5}, T_{2E5})$	94.999994	83.33362	95.833332	(T_{2E5}, T_{2S5})	74.390251	62.499865	66.666667	$(T_{2(o(m))5}, T_{2S5})$	70.6707333	52.0834006	63.888888
$(T_{2(o(m))6}, T_{2E6})$	95	83.333338	95.833334	(T_{2E6}, T_{2S6})	74.390244	62.499993	66.666667	$(T_{2(o(m))6}, T_{2S6})$	70.6707315	52.0833306	63.888889
$(T_{2(o(m))7}, T_{2E7})$	98.862815	83.333352	95.833333	(T_{2E7}, T_{2S7})	71.483633	62.500026	66.666667	$(T_{2(o(m))7}, T_{2S7})$	70.6707317	52.083366	63.888889
$(T_{2(o(m))8}, T_{2E8})$	95.000005	83.333277	95.833335	(T_{2E8}, T_{2S8})	74.390242	62.50004	66.666664	$(T_{2(o(m))8}, T_{2S8})$	70.670734	52.0833311	63.888888
$(T_{2(o(m))9}, T_{2E9})$	94.999998	83.333367	95.833334	(T_{2E9}, T_{2S9})	74.390245	62.5	66.666666	$(T_{2(o(m))9}, T_{2S9})$	70.6707309	52.0833546	63.888889
$(T_{2(o(m))10}, T_{2E10})$	94.999999	83.33759	95.833333	(T_{2E10}, T_{2S10})	72.241884	62.496822	66.666667	$(T_{2(o(m))10}, T_{2S10})$	68.6297895	52.0833451	63.888888
$(T_{2(o(m))11}, T_{2E11})$	95	83.333327	95.833334	(T_{2E11}, T_{2S11})	74.390244	62.500001	66.666667	$(T_{2(o(m))11}, T_{2S11})$	70.6707318	52.0833308	63.888889
$(T_{2(o(m))12}, T_{2E12})$	94.999708	8.3333333	95.833333	(T_{2E12}, T_{2S12})	74.390244	62.499998	68.085637	$(T_{2(o(m))12}, T_{2S12})$	70.6705149	5.20833321	65.248735
$(T_{2(o(m))13}, T_{2E13})$	94.999999	83.33335	95.833333	(T_{2E13}, T_{2S13})	74.390245	62.499973	66.666667	$(T_{2(o(m))13}, T_{2S13})$	70.6707324	52.0833209	63.88889
$(T_{2(o(m))14}, T_{2E14})$	95.000001	83.333244	95.833333	(T_{2E14}, T_{2S14})	74.390244	62.500021	66.666667	$(T_{2(o(m))14}, T_{2S14})$	70.6707321	52.0832949	63.888889
$(T_{2(o(m))15}, T_{2E15})$	95	833.33334	95.833334	(T_{2E15}, T_{2S15})	74.390244	6.25	66.666667	$(T_{2(o(m))15}, T_{2S15})$	70.6707318	52.0833339	63.888889
$(T_{2(o(m))16}, T_{2E16})$	95.000005	8.835771	95.680399	(T_{2E16}, T_{2S16})	74.39024	589.45992	66.666667	$(T_{2(o(m))16}, T_{2S16})$	70.6707316	52.0833288	63.786933

4.1 Simulation study

Simulation was carried out for T_{2Ej} , $T_{2[0(m)]j}$, T_{2Sj}, estimators using two accompanying variables (*X*, *Z*), when ranking is performed on Z. Multivariate random observations were generated from a trivariate normal distribution having parameters μ_X = 16, μ_Y = 12, μ_Z = 20, $\sigma_X^2 = \sigma_Y^2 = \sigma_Z^2 = 1$ and for different values of ρ_{XY}. The correlation coefficients between (*Y*, *Z*) and (*X*, *Z*) are assumed to be ρ_{YZ} = 0.9 and ρ_{XZ} = 0.80 respectively, with different sample sizes *m* = 3, 4, 5, 6, 7, 8, 9, 10 and *r* = 1.

5000 simulations were conducted to estimate the biases, MSEs, R.E, and P.R.E in order to ascertain the veracity of the theoretical underpinnings of this paper and to evaluate the performances of the proposed classes of estimators under ERSS over its corresponding counterparts based on SRS. The result of this simulation is presented in table 12.

TABLE 12

Simulation Results of MSEs, R.E, AND P.R.E of T_{2Ej}, $T_{2(o(m))j}$, T_{2Sj}, j=16

m	T_{1Ej}	$T_{1(o(m))j}$	T_{1Sj}	RE_1	RE_2	PRE_1	PRE_2
				ρ_{xy}=0.90			
3	1.404262	0.9420067	2.808467	0.50001017	0.33541667	50.0010167	33.5416667
4	1.401955	1.0558315	2.803868	0.50000763	0.3765625	50.0007625	37.65625
5	1.436618	1.1528716	2.8732	0.5000061	0.40125	50.00061	40.125
6	1.397857	1.1677815	2.795686	0.50000508	0.41770833	50.0005083	41.7708333
7	1.416604	1.2167506	2.833182	0.50000436	0.42946429	50.0004357	42.9464286
8	1.459227	1.2790943	2.918433	0.50000381	0.43828125	50.0003813	43.828125
9	1.419665	1.263888	2.839312	0.50000339	0.44513889	50.0003389	44.5138889
10	1.433852	1.2922512	2.867686	0.50000305	0.450625	50.000305	45.0625
m				ρ_{xy}= 0. 80			
3	1.270183	0.8188868	2.539774	0.50011667	0.3224251	50.0116667	32.2425096
4	1.228545	0.920252	2.456661	0.5000875	0.37459469	50.00875	37.4594693
5	1.227174	0.9796272	2.454005	0.50007	0.39919523	50.007	39.919523
6	1.224677	1.0870001	2.449068	0.50005833	0.44384235	50.0058333	44.3842351
7	1.304531	1.0812416	2.6088	0.50005	0.41445932	50.005	41.4459319
8	1.261559	1.106083	2.522897	0.50004375	0.43841782	50.004375	43.841782
9	1.264193	1.1266588	2.52819	0.50003889	0.44563856	50.0038889	44.5638555
10	1.26758	1.1378952	2.534982	0.500035	0.44887698	50.0035	44.8876975
m				ρ_{xy}=0.75			
3	1.144056	0.700946	2.376088	0.48148702	0.295	48.148702	29.5
4	1.144025	0.7921737	2.287866	0.50004013	0.34625	50.0040125	34.625
5	1.194183	0.900356	2.388212	0.5000321	0.377	50.00321	37.7
6	1.160379	0.9224521	2.320634	0.50002675	0.3975	50.002675	39.75
7	1.196098	0.9858815	2.392087	0.50002293	0.41214286	50.0022929	41.2142857
8	1.19188	1.0085879	2.383664	0.50002006	0.423125	50.0020063	42.3125
9	1.160928	1.0022322	2.321774	0.50001783	0.43166667	50.0017833	43.1666667
10	1.142525	1.0019625	2.284977	0.50001605	0.4385	50.001605	43.85
m				ρ_{xy}= 0.50			
3	0.719316	0.503484	1.438526	0.500037	0.35	50.0037017	35
4	0.715929	0.5548144	1.431779	0.500028	0.3875	50.0027763	38.75
5	0.750581	0.6154488	1.501095	0.500022	0.41	50.002221	41
6	0.729493	0.6200463	1.458933	0.500019	0.425	50.0018508	42.5
7	0.740651	0.6454038	1.481255	0.500016	0.43571429	50.0015864	43.5714286
8	0.736514	0.6536383	1.472988	0.500014	0.44375	50.0013881	44.375
9	0.724037	0.6516168	1.448037	0.500012	0.45	50.0012339	45
10	0.728649	0.6630559	1.457266	0.500011	0.455	50.0011105	45.5

5.0 Conclusion

A family of ratio-cum-product estimators of population mean of the study variable Y have been successfully proposed following information on two accompanying variables under ERSS as shown in equations (18) and (19) while keeping track record of the SRS version of the proposed estimators as shown in (20) for the purpose of efficiency comparison. Members of the proposed class of the estimators were obtained by varying the scalars that helps in designing the estimator and were presented in table 1, table 2, and table 3 respectively. Their properties such as biases, and MSEs were all derived as can be envisage in equations (28), (44), (51) for biases and (30),(45), (52) for MSEs. The Optimal Mean Square Errors were also derived to the quadratic polynomial form of Taylor's series approximation and presented in (43),(50) and (58) respectively. Theoretical underpinnings and the condition for which the proposed class of estimator would provide an appreciable gain in efficiency over its counterpart estimator were established and shown in (59).(60),(61),(62),and (63). Empirical and simulation studies were conducted to ascertain the veracity of the theoretical underpinnings of the work. From where it was discovered from the results that the proposed family of estimators based on ERSS provided smaller MSEs, R.E, P.R.E, for all values of the correlation coefficients and sample sizes considered in this paper and were therefore adjudged to be more efficient than the corresponding counterpart under SRS. This evidence is presented in table 7, table 8 , table 9, table 10, and table 11.

The efficiency of $\boldsymbol{T_{2Ej}}$, $\boldsymbol{T_{2(o(m))j}}$ $\boldsymbol{T_{2Sj}}$ increases for smaller values of correlation coefficient ρ_{XY} $_{=+}$*0.80, and +0.75*, and for smaller values of sample size and decreases for the values of the correlation coefficient ρ_{XY} =±*0.90* and *+0.50* and as the sample size increases in most cases in table 12.

The proposed estimators are approximately unbiased for all cases, correlation coefficients, and sample sizes considered in the simulation study.

The estimator $T_{2(o(m))j}$ performs better than that of T_{2Ej} and T_{2Sj} for all the values of the correlated coefficient and the samples sizes considered in this paper.

Therefore, the estimator*s* $T_{2(o(m))j}$ was adjudged to be the most efficient estimators among their brethren T_{2Ej} , T_{2Sj} since it produces the smallest MSEs in all the population, correlation coefficients, and sample sizes considered in this paper. The estimators in question were therefore adjudged to be efficient and provide a better alternative whenever efficiency is required.

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